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ABSTRACT

The question of developmental synchrony within the concrete operations period described in Piagetian literature was investigated. The idea of synchrony has been challenged by Brainerd's initial groupement research, which indicated a two-dimensional structure, corresponding to class operations and relational operations. A review of the literature indicated conflicting views concerning the nature of synchrony and how it may best be investigated. The secondary issue of distinguishing which groupements were operationalized by several traditional Piagetian tasks was explored. Indications from this study were (1) that the class-relational distinction was not a clear-cut one; (2) considering absolute difficulty level, groupements I, II, and III were more difficult than the other five groupement tasks; however, groupement IV, also a "class" task, indicated an intermediate difficulty level in the older grades; (3) traditional Piagetian performance does not appear to be completely subsumed by the groupement tasks; and (4) nonmetric scaling and clustering techniques aid in the analysis of dichotomous, developmental data. (Author/BJG)

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Technical Report No. 316

MULTIDIMENSIONAL SCALING OF PIAGETIAN TASK PERFORMANCE

by

Roberta Elaine Dihoff

Report from the Project on

Children's Learning and Development

Frank H. Hooper
Principal Investigator

U S DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

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TABLE OF CONTENTS

Acknowledgments iv

List of Tables vii

List of Figures ix

Abstract xi

I. Introduction 1

II. Review of Related Literature 3

The Logic of Concrete Operations 3
Properties of the Groupement 4
Classification 4
Seriation (Asymmetrical Transitive Relations) 5
Number 5
Empirical Research 6
Developmental Synchrony 6
Brainerd's Initial Groupement Research 9
Dimensional Studies 10

III. Statement of the Problem 11

IV. Methodology 13

Subjects 13
Assessment Tasks 13
General Procedure 15
Initial Methodological Considerations 15
The Techniques 15
Rationale 16
SSA 16
Numerical Taxonomy Package 16

V. Results 19

Correlational Data 19
Parametric Considerations 19
Principle Components Analyses 28
Reliability of the Groupement Tasks 28
Dimensionality 35
Multidimensional Scaling 35
Clustering 36
Composition - Reverse Operations 39
Traditional Piagetian Tasks 39

VI. Discussion 41

Initial Considerations 41
Developmental Synchrony 41
Methodological Procedures 42
Summary and Conclusions 42

References 45

Appendix A: Assessment Battery 49

Appendix B: Monotone Relationship of Gamma and Bentler's Coefficient 89

LIST OF TABLES

Table		
1	Frequency and Percentages of Subjects Passing Each Task by Grade	20
2	Number Passing Both Tasks by Grade (N=60)	21
3	Frequency and Percentages of Subjects Passing the Composition-Reverse Operations at Each Grade Level	22
4	Gammas and Bentler's Coefficients--Kindergarten and Third Grade Combined	23
5	Gammas and Bentler's Coefficients--All Grades	24
6	Pearson Product Moment Correlations for the Total Sample (Interval Scoring, N=180)	25
7	Relationship of Traditional Tasks to Groupement Performance, Part 1 (Gammas, N=120)	26
8	Relationship of Traditional Tasks to Groupement Performance, Part 2 (Gammas, N=120)	27
9	Means by Age, Sex, and School	29
10	Standard Deviations by Age, Sex, and School	30
11	Summary Table for Analysis of Variance (Total Sample, N=180)	31
12	Principle Components Analysis of the Kindergarten and Third Grade Subsample (N=120)	33
13	Principle Components Analysis of the Full Sample (N=180)	34
14	Composition-Inversion Relationship of Class Groupement/Composition-Reciprocal Relationship of Relations Groupement (N=180)	40

LIST OF FIGURES

Figure		Page
1	Two Dimensional Representation (N-180)	36
2	Two Dimensional Representation (N=120)	37
3	Multidimensional Stress Curves	37
4	Clustering of Bentler's m and gamma (N-120) (K,3)	38
5	Clustering--Interval Data/Gammas	38

ABSTRACT

The purpose of this study was to investigate the question of developmental synchrony within the concrete operations period described in Piagetian literature. The idea of synchrony has been challenged by Brainerd's initial groupement research, which indicated a two-dimensional structure, corresponding to class operations and relational operations.

A review of the literature indicated conflicting views concerning the nature of synchrony and how it may best be investigated. The secondary issue of distinguishing which groupements were operationalized by several traditional Piagetian tasks was explored.

Indications from this study were (1) that the class-relational distinction was not a clear-cut one; (2) considering absolute difficulty level, groupements I, II, and III were more difficult than the other five groupement tasks; however, groupement IV, also a "class" task, indicated an intermediate difficulty level in the older grades; (3) traditional Piagetian performance does not appear to be completely subsumed by the groupement tasks; and (4) nonmetric scaling and clustering techniques aid in the analysis of dichotomous, developmental data.

I

INTRODUCTION

Of central importance to the concrete operations period of Piagetian theory is the issue of the logical *groupements*, which underly the mastery of skills such as classification, seriation, and conservation. Although these constructs are hypothesized to exist and are inferred indirectly from performance that may utilize more than one groupement, little has been done to directly assess them as independent abilities. Recent work by Brainerd (1972a) attempting to measure each operation directly and independently has led to controversy concerning the feasibility of such an endeavor and has directly attacked the questions of developmental synchrony and sequencing. The purpose of this paper is to discuss the effectiveness of Brainerd's groupement tasks in assessing the eight groupings of classes and relations which comprise the middle childhood period in Piagetian theory.

Specifically Piaget deals with this period in terms of the following eight structures:

Classes

I Primary addition or union of classes; $(A + A' = B, B + B' = C)$

II Secondary addition or union of classes; (constructing series parallel to the initial one such that $A_2 + A_2' = B$)

III Bi-univocal multiplication of classes, denoting simple intersection of subordinate classes implying dual class membership, i.e., a bat exemplifies the intersection between a class of mammals and aerial animals (Wohlwill, 1966, p. 61)

IV Co-univocal multiplication of classes, denoting the intersection of nested subclasses.

Relations

V Addition of asymmetrical relations; $(A < B) + (B < C) = (A < C)$

VI Addition of symmetrical relations; $(A = B) + (B = C) = (A = C)$

VII Bi-univocal multiplication of relations; $(A < B < C$ in two dimensions)

VIII Co-univocal multiplication of relations; $(A < B < C$ on one dimension while $A = B = C$ on a second dimension)

The theory states that these eight structures each include a composition operation and either its inverse (if it is one of the four "classes" groupements) or its reciprocal (if it is one of the four "relational" groupements). The eight structures and the composition-inversion or

composition-reciprocity operations of each structure are hypothesized to emerge synchronously. Termed a "species specific" phenomenon, (Brainerd, 1972a), the synchrony of emergence may show individual variation; however, no invariant order of emergence is postulated for the group.

There is great controversy in the literature which deals with traditional Piagetian tasks concerning inconsistencies which indicate that certain operations emerge earlier than hypothesized or are independent of assumed prerequisite skills. Widely differing measurement instruments and scoring criteria (such as credence allocated to verbal explanations of performance) account for some of the inconsistencies; controversy continues over which procedures are correct (Brainerd, 1973a). Brainerd, in constructing the groupement tasks, contends that explanation criterion is "inappropriate to the task of determining the of cognitive structures, [Brainerd, 1972a, p. 10]," while the judgment is well-suited to this endeavor. Brainerd indicates that the group tasks reliably correspond to the logico-mathematical structures hypothesized by Piaget. However, a cross-sectional analysis, utilizing these tasks alone, indicates considerable divergence from theory, since it neither confirms the synchrony of emergence of the eight structures nor the simultaneous appearance of the composition operation and the reverse operation. Whether this is indeed an inconsistency within the theory or an artifact of the measurement instrument remains to be explored.

It is difficult to view mastery of a single task as developing in isolation and not as part of "an extensive and interlocking cognitive system [Wohlwill, Fusaro, & Devoe, 1969, p. 1]." Although the literature abounds with research on concept acquisition, we still have not begun to answer the question: "How do these concepts develop in the course of normal events, that is to say, given a child so educationally deprived as to have never wandered into a Piagetian experimenter (Wohlwill, Fusaro, & Devoe, 1969, p. 2)." Piaget concedes the importance of experimental influences, stating that the age norms he finds are specific to the environment in which he is testing. However, he too often assumes that mastery of a single task denotes the presence of a logical structure. The following discussion will be an attempt to put task mastery and logical structures into reasonable perspective.

II

REVIEW OF RELATED LITERATURE

THE LOGIC OF CONCRETE OPERATIONS

When describing the cognitive functioning of the middle childhood period, attempts to reconcile Piaget's logico-mathematical model with the observed intellectual functioning of the child lead to a major theoretical impasse, Piaget specifies that the logico-mathematical structures "constitute very good models of actual organization and processes of cognition [Flavell, 1963, p. 169]"; however, the existence of quantifiable psychological structures is not assumed (Piaget and Inhelder, 1969; Flavell 1963). For example, the existence of groupements IV and VI remains to be demonstrated empirically (Flavell, 1963). This distinction between the model and the psychological process is important. The relationship of the logico-mathematical structure to applied psychology is a difficult one, leading many psychologists to attempt to resolve practical considerations concerning how such structures may be verified. This endeavor leads to another instance of a competence vs. performance stalemate reached in many disciplines. "An enormous chasm exists between the theory of the groupement structure and the world of empirical tests, and Piaget rarely gives even the vaguest hints about how the chasm might be spanned [Brainerd, 1972a, p. 3]."

In order to examine the existence of psychological structures, it is first necessary to explore the logical structures, which "serve as the theoretical pattern formulated after thought has been constructed and not this living process of construction itself [Piaget, 1966, p. 31]." Bart (1973) contends that this distinction, due to the conceptual language of symbolic logic, eludes development psychologists.

> When it comes, however, to describing the structure of particular intellectual operations from the qualitative angle independently of their measurable (or metrical) performance, one has no option but to use, as a language, the general theory of structures which in mathematics goes by the name of general algebra and which includes the algebra of modern logic. In a case of this kind, one most naturally remains on one's guard to avoid a confusion between the psychological content (which in a sense constitutes a logic, since it concerns the intellectual operations of the subject, hence his logic) and the form used to describe it (which is also a logic, but of the kind formulated by algebraists) . . . This description of operational structures in terms of qualitative algebra requires constant care in order to avoid interference between the intellectual operations of the observed subject (e.g., the child at such and such a level), those of the psychologist who is observing him and above all those who are involved in the language, when it is that of logical or algebraic structures generally [Piaget, 1969, pp. 145-146].

Traditional Piagetian tasks have encountered the additional problem of confounding the structural components used to describe their mastery. For example, it may be postulated that groupements I and II are necessary to account for the partitioning of a stimulus array and are utilized in class inclusion tasks (Hooper, Sipple, Goldman, & Swinton, 1974); groupements I, II, III, and V may be necessary for the mastery of number tasks, seriation, and transitivity (Inhelder and Piaget, 1964; Hooper, et al., 1974); groupements VII and VIII may be necessary to account for serial correspondence and conservation (Piaget, 1952). It is also interesting to note that, although synchrony is the ideal, Piaget, in his empirical studies, has found a relative difficulty in abilities relying heavily on groupement I and the central importance of groupement V. These notions have found support in Brainerd's scalogram analysis (Piaget, 1952; Brainerd, 1972b). The relative difficulty of multiple classificatory skills (Lagattuta, 1970; Kofsky, 1966) also broaches the question of developmental synchrony.

PROPERTIES OF THE GROUPEMENT

It is necessary to elaborate on the formal properties of the groupement before discussing empirical validation of their structures. Five formal properties are included in each structure: (1) a composition operation; (2) the reverse of the composition operation; (3) the general identity element, i.e., if the operation is a class union, the element is the null set; (4) special identity ("tautologies") or ordering property; (5) associativity such that "the end result of any set of composition operations is independent of the order in which the specific operations are carried out [cf., Flavell, 1963, p. 89]." In the empirical investigations to be carried out, the first two of these properties are of major importance.

CLASSIFICATION

Inhelder and Piaget (1964) describe classes in terms of their intensive and extensive properties. Early class relations indicate that items grouped by usage rather than in hierarchical relationships and that they show imperfect quantification. In order for mastery of classes to be operational, the child should be able to define a class intensively by relating it to a more general class, distinguishing it from other classes, and using quantifiers extensively. This implies the use of union, intersection, and complement of classes and is normally not mastered until the child is 9-10 years old (Inhelder & Piaget, 1964).

Primitive classifications can be performed perceptually. The following indicates the emergence pattern of classificatory abilities: (1) "figural collections"; (2) forming piles and subcollections without the ability to compare them quantitatively; and (3) hierarchical classification, including quantifying inclusion. Tasks requiring the comparison of a superordinate and subordinate set without being concerned with the actual number of items are termed "some-all." They would be comprised of such relationships as all dogs are mammals but not all mammals are dogs. Class inclusion tasks typically compare subset A to set B.

Brainerd and Kaszor, (1974) suggest that the contention that subjects compare subset A to subset A' is without empirical substantiation.

SERIATION (ASYMMETRICAL TRANSITIVE RELATIONS)

A second important operation of the middle childhood period is seriation or relationality. Piaget states that as the child learns additive seriation, he immediately understands multiplicative seriation in the form of correspondence (Piaget, 1966). The three stages of seriation are described as follows: (1) no seriation; (2) trial and error, at which stage the subject will seriate the entire array before inserting a missing stick; (3) systematic seriation, making comparisons from only one end of the array.

Inhelder and Piaget (1964) discuss operational considerations concerning the perceptual influence on relational tasks. The perceptual array is before the child as he makes his comparisons. Since performance relies heavily on experience, perception is not the sole means of discrimination. However, it does provide an auxiliary mode of functioning that is not present in classificatory behavior. Although operative seriation is not postulated to be mastered before operative classification, intermediate stages show differences. The differences may be in the mode of functioning since quite often in Piagetian tasks it is not the correct solution but the process by which the solution is reached that indicates operationality.

NUMBER

Piaget's position is that the acquisition of classification and seriation skills also entails the advent of a number system. Although the young child has an understanding, either perceptual or rote, of number, in order to have a logical, manipulative concept of number he must have these operations (Piaget, 1966).

Recently the logical and functional nature of number has come under close scrutiny. Piaget and Inhelder (1969) recognize three hypotheses that have gained support: (1) number is independent of logical structures; (2) it derives directly from them (cardinal number arising from classes and ordinal number from seriation); (3) "it constitutes a new and original synthesis; all elements of this synthesis are borrowed from 'grouping' structures but the total structure results from a new mode of composition [Piaget and Inhelder, 1969]." The first of these views is preferred by the intuitionists; the second is from Whitehead and Russell, and the third is considered the most appropriate by Piaget and Inhelder (1969). Brainerd (1973b) describes this conflict in terms of an ordinal-cardinal conflict, which followed from the general dissatisfaction with the intuitionist viewpoint. The intuitionist point of view followed Pythagorean theory that number, which underlies all mathematics, was beyond the understanding of man. As this was discarded, mathematicians split to adhere either to an ordinal view, which stresses ordered progressions; or a cardinal view, which stresses that numbers can represent manyness. While Piaget argues that an understanding of number is contingent upon both of these concepts, Brainerd supports the ordinal view (Brainerd, 1973b).

EMPIRICAL RESEARCH

Various aspects of both classification and seriation rely on additive operations affecting classes and relations. Operations relying on several seriations or classifications are multiplicative, i.e., serial correspondence. Mastery of these structures is inferred by the ability to exhaustively and consistently sort items, subdivide sets into subsets, and understand hierarchical relationships. Traditional tasks investigating these skills have included matrix, some-all, and class inclusion tasks; the latter two are used in the present study.

DEVELOPMENTAL SYNCHRONY

In a cross-sectional comparison of classificatory and seriation skills, Lagattuta (1970) found that children from 5 1/2 to 6 1/2 years of age could perform simple and multiple classification tasks successfully. At that age children could begin to perform successfully on simple seriation tasks while a serial matrix could not be successfully completed until 8 1/2 years of age. The conclusion was that classification and seriation skills develop independently.

The literature dealing with the investigation of the interrelationship of class and relational abilities is generally sketchy and incomplete leading to unsatisfactory conflicts, probably due in part to inconsistencies in task formats. Reporting findings consistent with those of Inhelder and Piaget (1964), Lovell, Mitchell, and Everett (1962), Shantz (1967), and Smedslund (1964) have found notable degrees of interrelationship. However, an impressive number of studies have challenged the Genevan position, Brainerd (1972a), Chittenden (1964), Lagattuta (1970), Berzonsky (1971), Nassafat (1963), and Wohlwill, Fusaro, and Devoe (1969).

Smedslund (1964), using 160 subjects from 4 to 11 years old, found support for a higher degree of difficulty for multiple classification and relationality than class inclusion. The difficulty levels of multiple classification and relationality were fairly comparable. These contradictory findings are ambiguous with respect to sequence and synchrony as discussed by Inhelder and Piaget (1964).

Investigating classificatory abilities of children aged 5 to 8 years of age, Wohlwill, Devoe, and Fusaro (1969) administered tasks assessing grouping, class intersection, and class inclusion, three times over an 18-month span. With age, the grouping task indicated increased dependence on categorical rules and the class intersection task showed significantly more successful completions, with an increasing number of dimensions verbalized. There were no consistent differences found for the class inclusion tasks, although the pictorially presented tasks were more difficult than the verbally presented ones.

Gildemeister (1974), utilizing matrix tasks, found that concrete operational children can abstract "relevant task information"; however, conceptual and perceptual dimensions specific to the task and stimulus set do affect performance.

These minor asynchronies are explained by Inhelder and Piaget (1964) as follows:

> (1) Children reach an operational level in the multiplication of series about the same period as cross-classification. (2) Nevertheless the first of the schemata does entail a problem of its own, a problem of spatial symbolism and not one of logical structure. From the age of 7 or 8, on average, they show that they understand the need to observe the equivalences as well as the differences involved.
>
> But they do not all hit on a two-dimensional symbolism right away; some do, but others use a one-dimensional cyclic order.
>
> Finally there are four principal groupings in the logic of classes and relations, corresponding with simple and multiple classification and simple and multiple seriation. It is a most remarkable fact that, in spite of the differences noted in respect of ease of perceptualization, all four structures become operational at roughly the same period. There are certain minor differences depending on the extent to which the content of a problem lends itself to imaginal representation, but they do not invalidate our main theses [p. 278-279].

Kofsky's authoritative study (1966) was designed to investigate the acquisition of classificatory skills. The children, aged 4 to 9 years of age, were given eleven tasks designed to differentiate the steps in which classification develops. A unidimensional scalogram technique was employed to judge how many children passing a task had passed all the theoretically prerequisite tasks. Significant age differences were found between the nine-year olds and the seven- and eight-year olds and between the seven- and eight-year olds and the younger subjects. A partial order of emergence consistent with the stages previously mentioned indicated the following pattern: (1) mastery of consistent sorting and resemblance sorting (figural collections); (2) some-all and exhaustive classes; (3) elementary relations among object and classes in hierarchy; (4) conservation of hierarchy; (5) inclusion; (6) hierarchical classification. An acceptable index of reproducibility (.93) was achieved by deleting certain tasks, i.e., conservation, exhaustive sorting, and inclusion tasks; however, since Torgerson suggests a minimum of ten items for scaling dichotomous data, these deletions may have been ill-advised (Torgerson, 1958). Again, performance variables mentioned previously with respect to the some-all and inclusion tasks may have produced task-specific errors.

Piaget and Inhelder (1969) maintain that concrete operations are not content-free and thus are subject to variability due to stimulus differences. This was demonstrated when thirty children, aged 5 through 9, were asked class inclusion questions using both beads and flowers. Better performance was elicited from the younger children with flowers than with beads. In replicating this experiment Lovell, Mitchell and Everett (1962) also found variance in performance due to the material used; however, in this instance, the beads elicited more correct responses. Large developmental lags have been reported by Inhelder and

Piaget (1964) when pictures of animals were used as classificatory stimuli. Their rationale was that there was a larger degree of familiarity for beads and flowers. A similar distinction was found by Stephens (1972); however, with a group of five-year olds, Kohnstamm (1963) found no indication of a stimulus difference. Note that these children were much younger than those normally expected to be able to complete a class inclusion task, i.e., about 7 or 8 years of age.

In order to investigate the close conceptual relationship that many have inferred between the groupement structures and the number concept, much research has been initiated attempting to relate seriation, classification or conservation to number concepts. Elkind (1964) investigated the relationship between seriation and number with three groups of children, aged four, five, and six years old. The tests, conducted with size-graded blocks, slats, and sticks, consisted of the discrimination of the largest and smallest sticks (or slats or blocks), seriation of the stimulus materials, an insertion of a "missing" object into the constructed array, and ordinal-cardinal correspondence. The ANOVAs involving three independent variables (age, materials, and tests) were all significant. The only interaction falling below the .01 level involved materials; however, these were still significant at the .05 level, indicating material-specific responses in this area also. The seriation stages, specifically (1) perceptual (picking the largest or smallest stick), (2) trial and error seriation, and (3) abstraction, were confirmed. Viewing the development of number as a coordination of asymmetric (series) with symmetric (class) relations, Elkind postulated a conceptual unity between seriation, classification, and number, which he also concluded developed in three parallel stages.

> When this system of operations is applied to elements regarded as similar, the result is classification; when the same system of operations is applied to elements regarded as different, the result is series. When the same system of operations is applied to elements regarded as both alike and different, the result is number. However, in contrast to the elements of classes and series, the elements of number are constructed by the child's own actions and are not given in immediate perception or in intuition [Inhelder and Piaget, 1969, p. 197].

Wohlwill (1960), using scalogram techniques, reaffirmed three stages in the development of number. His study was conducted with four-, five-, and six-year olds and involved tasks with various degrees of perceptual cues and higher order principles of conservation and ordinal-cardinal correspondence. Inferring from his results a single developmental process proceeding toward increased symbolic mediation, Wohlwill discusses the difficulty of making developmental inferences with this type of technique. After validating the procedure, he discusses its implications--such as discovering the origin of such a sequence.

Scaling techniques were further employed by Dodwell (1962) in her study which led to the assumption that hierarchical classification and cardination skills develop independently. The subjects were sixty children between the ages of five and eight, in four different grades. Although the concepts were mastered within the same age range, no clear relationship was observed, indicating support for Piaget and Inhelder's (1969) premise that number is a new synthesis. As Piaget (1952) elaborates:

> Hitherto, we have considered number as a seriated class, i.e., as the product of class and asymmetrical relation. But this in no way implies that class and asymmetrical relation come before number. On the contrary, number can be regarded as being necessary for the completion of truly logical structures . . .. Instead of deriving number from class, or the converse, or considering the two as radically independent, we can regard them as complementary, and as developing side by side although directed towards different ends [p. 161].
>
> Class and number are mutually dependent, in that while number involves class, class in its turn relies implicitly on number [p. 184].

Thus, contrary to Dodwell's assumptions, it does not appear to be an instance in which unidimensional scaling is appropriate. The model which is being described does not postulate an invariant sequence but rather describes an interactive model which could show great variation in a cross-sectional analysis.

In Brainerd and Fraser's (1973) studies, the number concept has been treated alone, in an attempt to find support for their postulated ordinal theory of number. Using five- to seven-year olds with a sample size of 360 (and a later replication experiment with a sample of 100) they found the following order of difficulty: (1) ordination, (2) natural number (number conservation), (3) cardination. The results, as noted by Brainerd, do not indicate whether this is a result of lack of fundamental concepts or a greater emphasis on performance factors, such as memory.

In an early attempt to treat Piagetian tasks in terms of their structural components, Shantz (1967) hypothesized that multiplicative processes (classes, relations, and spatial relations) would be highly intercorrelated. Using the Raven to assess multiplication of classes, a multiple relations test constructed to assess the multiplication of asymmetric logical relations, and Piaget's landscape task, she administered her tasks to 24 children (12 male, 12 female) at each of three age levels, 7 1/2, 9 1/2, and 11 1/2. Her tasks showed significant correlations at 7 1/2 ($r_w = .56$, $p < .05$) and 9 1/2 ($r_w = .62$, $p < .01$) but not at 11 1/2 ($r_w = .51$, $p < .10$). These results would not be particularly surprising in view of Brainerd's (1972a) recent research of the groupement structures. The relative difficulty of classes over relations, which appeared to continue to be demonstrated in the performance of older children, could account for a portion of the variability. It is Brainerd's contention that classes and relations do not develop in synchrony but are a product of two structures, one corresponding to classes and the other relational.

BRAINERD'S INITIAL GROUPEMENT RESEARCH

It is Brainerd's contention that, in the four class groupements, Piaget is concerned with relationships between subordinate and superordinate classes and that the relational groupements refer to quantitative equivalence and difference relationships. In order to examine the class groupements (I-IV), colored circles and triangles were used to distinguish the sets and subsets, while colored sticks of differing

length and weight were used for the relational groupements (V-VIII). Simple judgments without explanations were used for scoring purposes.

A unidimensional scalogram analysis yielded the following sequence: (1) V & VI, (2) VII, (3) VIII, (4) IV, (5) III, and (6) I & II. The four relational groupements were found to appear about two-and-a-half years earlier than the four class groupements. It was also found that in the case of the class groupements the inverse operation was acquired earlier than the composition operation. Both of these findings are contrary to the idea of synchrony in Piagetian theory. A subsequent multidimensional technique indicated the possibility of a two dimensional construct, i.e., classes and relations (Brainerd, 1972b).

DIMENSIONAL STUDIES

The major problem with many of the dimensional analyses that consider Piagetian task performance is that they have been concerned primarily with other behavioral areas, such as moral reasoning (Stephens, 1972), environmental factors (Vernon, 1965), language (Lunzer and Wilkinson, 1973), memory, concept development, various types of reasoning, and I.Q. (Berzonsky, 1971), and creativity (O'Bryan & MacArthur, 1969). One would expect that measures of the same content area would, by definition, correlate more highly with each other than with measures of other content areas (see Wohwill, 1973; Campbell & Friske, 1959). It is therefore not surprising that studies that use these measures of similarity, such as clustering and factor analysis, have fairly consistently identified factors that are consistent with content areas. O'Bryan and MacArthur (1969) further analyzed the Piagetian battery to indicate a separate factor structure for inversion and reciprocity.

Studies dealing solely with Piagetian performance found distinctions (Hooper, et al., 1974; and Winkelmann, 1973). Hooper, et al. identified minimal differences between reproduction and transposition matrix tasks. In subjecting the tetrachoric correlations on entire task array to principle components analysis, they identified three major components, with class-inclusion and combinatorial reasoning loading highly on the first dimension. They concluded that a unidimensional pattern was not found due to the extreme range of item difficulty. Deleting the easiest items, class inclusion and combinatorial reasoning still loaded highly together and appeared the most difficult items in a Guttman Scalogram analysis. It was concluded that performance variables may significantly mask underlying structure. Winkelmann (1973) identified four conservation factors corresponding to conservation of substance (equality), conservation of substance (inequality), conservation of number (equality), and conservation of number (inequality).

III

STATEMENT OF THE PROBLEM

It is clear from the literature that the issue of synchrony is not one that can easily be resolved. Within the context of a well defined, fairly standardized set of tasks, the first issue is to compare the overall performance on class and relational tasks. If it is correct to assume that mastery of Brainerd's tasks constitutes mastery of the underlying structures of the concrete operations period, then the interrelationships of these tasks with the conventional Piagetian tasks used to assess mastery of the same underlying structures is of interest. Although it is questionable whether any practical skill is dependent upon a single groupement structure, and although it is assumed that many conventional tasks rely on several structures, it would be interesting to note whether a task is more highly related to a class or to a relational dimension--if indeed this distinction is confirmed.

It is therefore the purpose of this study to examine: (1) whether the results indicate concurrence with the theoretical view of synchrony (Piaget, 1966; Flavell, 1963) or reaffirm Brainerd's (1972a, 1972b) findings; and (2) how Brainerd's groupement tasks relate to the conventional tasks associated with the concrete operations period.

IV

METHODOLOGY

SUBJECTS

One hundred and eighty students from four schools in the Beloit (Wisconsin) School System participated in this study. These students are currently participating in a three-year, sequential study in conjunction with the Wisconsin Research and Development Center for Cognitive Learning in Madison, Wisconsin. Sixty subjects, thirty males and thirty females, were in each of three grade levels: kindergarten, third and sixth. The entire subject sample received the groupement tasks. Due to ceiling effects, however, not all received all of the conventional tasks. Therefore, a collapsed sample of kindergarten and third grade subjects, as well as a collapsed sample of kindergarten, third, and sixth grade subjects, was utilized in the analysis of some of the traditional Piagetian tasks.

ASSESSMENT TASKS[1]

1. Groupement Tasks (Adapted from Brainerd, 1972a)

There were eight groupement tasks, one corresponding to each of the eight logical structures. Each task contained eight questions, four on the composition operation and four on its reverse operation. The stimulus materials used for the four class groupements were pictorial representations of colored circles and triangles. The stimulus materials used for the four relational groupements were three colored sticks--one blue, one green, and one red. They varied in length and weight. Each subject was given the entire set of tasks. Scoring was done both intervally and dichotomously. Intervally, each subject received a score of 0-8, one point corresponding to a correct response on each question. A pass score was assigned if the subject correctly answered three out of four of the composition questions and three out of four of the questions concerning the reverse operation.

2. Dichotomies (Adapted from Kamii and Peper, 1969)

The stimulus set was a deck of cards with pictures which varied according to the shape, number, and color. The subject was required to consistently sort the cards on each of these dimensions over three trials to receive a "pass" score. Only the kindergarten and third grade subjects received this task.

3. Seriation Task Array (Adapted from Elkind, 1964)

There were four tasks administered to the kindergarten and third grade subjects consisting of seriating four sticks and then seven sticks to receive one "pass" score. Insertion of three missing sticks to form

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[1]See Appendix A for a detailed task description

a ten stick array constituted a second "pass." Requirements for a third "pass" consisted of placing this array into correspondence with seriated circles. The final task required the subject to retain this correspondence despite deformations of the stick array and justify his reasoning.

4. Number (Adapted from Brainerd, 1973b, & Brainerd and Brainerd, 1972)

The kindergarten and third grade subjects were tested on two number concepts, conservation of number and cardinality. For conservation of number, the experimenter and subject each formed a row of chips, placed in correspondence. The subject was asked three prediction questions concerning the quantity of chips if one row were pushed together. He was then asked the same three questions concerning the number of chips with the deformation performed. The rows were then returned to correspondence and one chip removed. The subject was questioned concerning the equality of the two rows. All questions had to be answered correctly without verbal explanations for a "pass." A justification was included but did not bear on the scoring procedure.

Cardinality I and II required a comparison of the number of dots in two rows, which were often perceptually unequal. All the questions had to be answered correctly for a "pass."

5. Combinatorial Reasoning (Adapted from Goodnow, 1962)

All 180 subjects received this task which consisted of forming all possible pairs of colors from eight different colored chips without any repeats. A perfect score of 28 correct pairs was required for a "pass."

6. Some-All (Adapted from Kofsky, 1966)

The kindergarten and third grade subjects received this task consisting of four questions asked with respect to picture representations of colored geometric forms. The questions were designed to investigate gross comparisons phrased with the word "all." Again, a strict scoring criterion was imposed and perfect performance was required for a "pass."

7. Transitivity (Length and Weight) (Adapted from Brainerd, 1973c)

The stimulus for transitivity of length consisted of a board with two sticks glued two feet apart. One stick was slightly shorter. The subject was asked to compare these two sticks after seeing each compared with a third stick, which was the same length as the longer stick. All the questions had to be answered correctly for a "pass."

The stimulus for transitivity of weight consisted of a red ball and a gray ball of equal weight and a gray ball of lighter weight. The subject had to compare each of the gray balls with the red one. He was then asked questions concerning the weight of each of the gray balls. All the questions had to be answered correctly for the subject to receive a "pass."

8. Conservation (Length and Width) (Adapted from Hooper (1969) and Brainerd (1973c)

These tasks were also only administered to the kindergarten and third grade subjects. Conservation of length included two formats, identity and equivalence. The identity format consisted of prediction and deforamtion questions based on the consequences of making a string into a circle. The equivalence format consisted of questioning the

subject concerning making one string into a circle without altering another string of equal length. All questions, without explanations, had to be answered correctly for a "pass."

Conservation of weight included two task formats, identity and equivalence. The identity stimulus was one clay ball. Questions concerned the consequence of changing the shape of the clay. The equivalence stimulus consisted of two clay balls of the same weight. Questions concerned the consequences of changing the shape of one of the clay balls with respect to altering the weight equivalence of the two pieces of clay. All the questions without explanations had to be answered correctly for a "pass."

9. Class Inclusion (Adapted from Kofsky, 1966)

The task was administered to all subjects. The stimulus consisted of three pictorial representations of geometric forms. Quantifiers (e.g., more) were used to compare sets and subsets. All questions had to be answered correctly for a "pass."

GENERAL PROCEDURE

Due to the extensive task battery, of which the groupement tasks are only a part, the third graders were tested in three sittings and the kindergarteners in four sittings. The sixth graders received the reduced battery in one sitting. The groupement tasks were administered in their entirety in one sitting and the order of presentation was not varied--"classes" always preceded "relations." The total testing time per session was fifty minutes, and each tester gave the entire battery to his subject. All of the testing was done individually.

INITIAL METHODOLOGICAL CONSIDERATIONS

When analyzing Piagetian data two considerations are of major importance. First, Piagetian research utilizes dichotomous performance criteria, so it is of major importance that this data be dealt with in the most meaningful way. Second, because the questions that are raised do not deal solely with interrelationships of dependent and independent variables but rather are concerned with invariant sequencing of behaviors determined not by age but by prerequisite skills, parametric techniques may be invalid to both the data and the questions that the experimenter wishes to examine.

THE TECHNIQUES

Since our data is dichotomous, it will be considered nonmetric. It may also be considered proximity data since we are attempting to determine "the number of factors or dimensions necessary to account for similarities [Subkoviak, 1972]." The measure of proximity chosen for the majority of the analyses was gamma. Gamma was the correlation of preference since it is unaffected by the marginal values and thus relatively insensitive to task difficulty level, (Goodman & Kruskal, 1954). However, given the two-choice type of questions and the six out of eight passing criterion, guessing can radically affect the magnitude of the correlation.

Also, the presence of a zero b or c cell spuriously yields a perfect correlation of 1.00 or -1.00. For this reason, tables with any small cells must be interpreted cautiously. A multidimensional scaling program (SSA) and a clustering package (Numerical Taxonomy Package) were utilized to attempt to replicate Brainerd's (1972b) class-relation distinction.

RATIONALE

Unidimensional scaling techniques have been used with limited success to qualitatively describe emergence patterns of concrete operational skills (Brainerd, 1972b; Dodwell, 1962; Hooper, et al.,1974; Kofsky, 1966; Wohlwill, 1966). Parametric factor analytic techniques have succeeded in bringing the idea of single operations and synchrony into question with little definitive success (Berzonsky, 1971; Hooper, et al., 1974) and major methodological difficulties. When dealing with a variety of conceptually divergent operations which are encompassed under the notion of classes and relations, the intuitive idea that mastery of these skills may require different abilities is not absurd.

SSA

Since the data can be viewed as proximity data and nonmetric, Torgerson's SSA program is applicable. This type of program is based on the following underlying assumptions: (1) that the tasks are based on the same model ("one that assumes a monotonal relationship between interpoint distances and given data [Shephard, 1972b, p. 22]"); (2) that an iterative procedure of adjusting coordinates for the points can/should be used to achieve a closer approximation of the desired monotonic relationship; and (3) that it is consistently reproducible when applied to the same data.

Through an iterative process, the program readjusts the points representing the eight objects until the best fit for the objects is achieved. A measure called stress indicates how good the fit is. Due to the small number of objects, solutions were attempted in one, two, and three dimensions only. The greater the number of dimensions in relation to the number of objects, the smaller the expected stress value. Thus, eight dimensions would yield no stress at all.

NUMERICAL TAXONOMY PACKAGE (BAKER, 1972)

Given an n x n matrix of proximity values, in the case of the groupement tasks of an 8 x 8 matrix of gammas, this program utilizes a hierarchical clustering system to find a substantively meaningful sequence of partitions based on these eight items. Operationally it begins with eight mutually exclusive single member groups and combines the pair with the highest proximity value. It continues combining partitions with the highest proximity until one all-inclusive group is formed. The final level or any intermediate level may be considered the most suitable. A helpful guideline for choosing the desirable level can be made by

inspecting a gamma value computed for each level and comparing this value with tables based on the number of objects to be clustered. The overall gamma value is considered by measure of randomness within clusters and is computed by listing all the possible sets of object pairs, 1,. . . . . ., N, assigning each pair a proximity rank of 1 to N and comparing this proximity rank to their partition rank, corresponding to the first level in which the pair appeared together. Gamma then equals the proportion of consistent rankings minus the proportion of inconsistent ranking or $\gamma = \frac{S+ - S-}{S+ + S-}$. Gamma values for intermediate levels are tabled (see Hubert, 1973, 1974). As suggested by Shephard (1972a, 1972b) this procedure is most useful in interpreting a scaling solution in more than one dimension in that it helps to locate the most meaningful rotation of the axis.

V

RESULTS

In the current study, subjects were drawn from three grade levels. As indicated by the percentage of subjects passing each task and the percentage of subjects passing one task while failing the other (Tables 1 and 2), groupements 1 and 2 were extremely difficult. They were passed at less than a level of random guessing, with the composition operation appearing to be extremely difficult (Table 3). A ceiling effect was indicated for the third and sixth grade subjects on groupement 6, with no third grade subjects passing any of the class groupements failing groupement 6. Since either of these two conditions would lead to spurious correlation coefficients in a two x two contingency table, age was collapsed to provide more reasonably sized cells.

CORRELATIONAL DATA

For the eight groupement tasks under initial investigation data were obtained both dichotomously and intervally. For each pair of the eight tasks scored dichotomously, two x two contingency tables were constructed and gammas computed for each age group, a combined kindergarten and third grade subsample, and an all grade sample (Tables 4 and 5).

In order to use the correlations for dichotomously scored data within a parametric framework while still preserving the rank order of the values, Bentler's coefficients of monotonicity were also calculated. Intervally scored data were analyzed using Pearson product moment correlations (Table 6). Differences between this correlational technique and the gammas in a dimensional analysis indicate that this correlation most strongly replicates the class-relation split indicated by Brainerd (1972b); however, the advisability of this correlation is doubtful since it relies heavily on marginal values and is thus affected by task difficulty.

Since the remainder of the task array was only scored dichotomously, only gammas were computed for the interrelationship between these tasks and the groupement (Tables 7 and 8). Since the entire subject sample did not receive the entire battery, most of the tasks were only related to the kindergarten and third grade combined subsample. Despite the wide range of performance levels subsumed by the groupement tasks, it appears the traditional Piagetian tasks, most notably the conservation tasks and the number tasks, have task-specific properties not assessed by the groupement array.

PARAMETRIC CONSIDERATIONS

Interval scores were based on a scale of 0 to 8 where the subject received one point for a correct answer on each of the eight questions

TABLE 1

FREQUENCY AND PERCENTAGES OF SUBJECTS PASSING EACH TASK BY GRADE

		I	II	III	IV	V	VI	VII	VIII
K	#	8	12	4	8	32	40	27	20
	%	13.33	20.00	6.67	13.33	53.33	66.67	45.00	33.33
3rd	#	5	5	8	17	42	56	40	33
	%	8.33	8.33	13.33	28.33	70.00	93.33	66.67	55.00
6th	#	25	28	16	35	48	54	46	48
	%	41.67	46.67	26.67	58.33	80.00	90.00	76.67	80.00

Number Passing Both Tasks (N=180)

	I	II	III	IV	V	VI	VII	VIII
I								
II	25							
III	15	14						
IV	28	25	18					
V	29	31	25	51				
VI	30	35	26	57	107			
VII	26	30	20	45	88	103		
VIII	28	31	22	47	81	94	79	

Number Passing Both Tasks (N=120)

	I	II	III	IV	V	VI	VII	VIII
I								
II	6							
III	4	4						
IV	7	5	5					
V	8	11	10	22				
VI	8	10	10	23	63			
VII	7	9	7	17	50	60		
VIII	6	8	8	17	41	50	41	

TABLE 2

NUMBER PASSING BOTH TASKS BY GRADE (N=60)

		I	II	III	IV	V	VI	VII	VIII
Sixth Grade	I								
	II	19							
	III	11	10						
	IV	21	20	13					
	V	21	20	15	29				
	VI	22	25	16*	34	44			
	VII	19	21	13	28	38	43		
	VIII	22	23	14	30	40	44	38	
Third Grade	I								
	II	3							
	III	3	3						
	IV	4	3	4					
	V	3	4	7	16				
	VI	5*	5*	8*	17*	39			
	VII	3	3	5	12	31	39		
	VIII	3	5*	6	14	27	31	26	
Kindergarten	I								
	II	3							
	III	1	1						
	IV	3	2	1					
	V	5	7	3	6				
	VI	3	5	2	6	24			
	VII	4	6	2	5	19	21		
	VIII	3	3	2	3	14	19	15	

*No one passing column task failed row task.

TABLE 3

FREQUENCY AND PERCENTAGES OF SUBJECTS
PASSING THE COMPOSITION-REVERSE OPERATIONS AT EACH GRADE LEVEL

		I		II		III		IV	
		C	I	C	I	C	I	C	I
K	#	2	39	5	42	2	13	5	17
	%	3.33	65.00	8.33	70.00	3.33	21.67	8.33	28.33
3rd	#	3	54	4	55	5	21	15	40
	%	5.00	90.00	6.67	91.67	8.33	35.00	25.00	66.67
6th	#	12	58	20	60	11	26	27	48
	%	20	96.67	33.33	100.00	18.33	43.33	45.00	80.00

		V		VI		VII		VIII	
		C	R	C	R	C	R	C	R
K	#	20	38	26	42	17	24	11	16
	%	33.33	63.33	43.33	70.00	28.33	40.00	18.33	26.67
3rd	#	34	51	55	57	32	37	23	35
	%	56.67	85.00	91.67	95.00	53.33	61.67	38.33	58.33
6th	#	48	54	53	55	36	41	36	43
	%	80.00	90.00	88.33	91.67	60.00	68.33	60.00	71.67

TABLE 4

GAMMAS AND BENTLER'S COEFFICIENTS
KINDERGARTEN AND THIRD GRADE COMBINED

		I	II	III	IV	V	VI	VII	VIII
Gammas	I	1.00							
	II	.76	1.00						
	III	.69	.57	1.00					
	IV	.70	.27	.52	1.00				
	V	-.00	.08	.55	.72	1.00			
	VI	-.49	-.56	.12	.55	.39	1.00		
	VII	-.04	-.07	.06	.31	.56	.60	1.00	
	VIII	.04	.07	.47	.56	.56	.77	.69	1.00

		I	II	III	IV	V	VI	VII	VIII
Bentlers	I	1.00							
	II	.47	1.00						
	III	.40	.31	1.00					
	IV	.41	.14	.28	1.00				
	V	-.00	.04	.30	.42	1.00			
	VI	-.26	-.31	.06	.30	.20	1.00		
	VII	-.23	-.03	.03	.16	.31	.34	1.00	
	VIII	.23	.03	.25	.30	.31	.47	.40	1.00

TABLE 5

GAMMAS AND BENTLER'S COEFFICIENTS
ALL GRADES

		I	II	III	IV	V	VI	VII	VIII	Comb. R.	Class Incl.
Gammas (dichotomous data)	I	1.00									
	II	.84	1.00								
	III	.73	.59	1.00							
	IV	.81	.56	.65	1.00						
	V	.26	.03	.65	.59	1.00					
	VI	-.19	-.24	.49	.69	.43	1.00				
	VII	.16	.11	.23	.39	.55	.63	1.00			
	VIII	.45	.35	.54	.63	.58	.69	.65	1.00		
	C.R.	.63	.58	.60	.79	.48	1.00	.49	.41	1.00	
	C.I.	.58	.56	.53	.82	.31	.80	.39	.59	.67	1.00

		I	II	III	IV	V	VI	VII	VIII
Bentlers	I	1.00							
	II	.55	1.00						
	III	.44	.33	1.00					
	IV	.51	.31	.37	1.00				
	V	.13	.02	.37	.33	1.00			
	VI	-.09	-.12	.26	.40	.22	1.00		
	VII	.08	.06	.12	.21	.30	.35	1.00	
	VIII	.24	.18	.27	.36	.32	.40	.37	1.00

TABLE 6

PEARSON PRODUCT MOMENT CORRELATIONS FOR THE TOTAL SAMPLE
(INTERVAL SCORING) (N=180)

	I	II	III	IV	V	VI	VII	VIII
I	1.00							
II	.559	1.00						
III	.259	.268	1.00					
IV	.255	.273	.313	1.00				
V	.069	-.048	.195	.196	1.00			
VI	.077	.073	.035	.263	.150	1.00		
VII	.101	.059	-.030	.106	.252	.179	1.00	
VIII	.116	-.059	.202	.094	.256	.344	.386	1.00

TABLE 7

RELATIONSHIP OF TRADITIONAL TASKS TO GROUPEMENT PERFORMANCE, PART 1
(GAMMAS, N=120)

	I	II	III	IV	V	VI	VII	VIII
Serial Corresp.	.23	.33	.28	.60	.59	1.00	.49	.66
Seriation 1	-.38	-.25	.52	.47	.01	.59	.25	.32
2	.68	.35	-.64	-.25	-.36	-.64	-.50	-.61
3	-.44	.14	-.02	.02	.30	.50	.30	.11
Dichotomies	.10	.30	.38	.66	.59	.76	.38	.44
# Conservation	-.25	-.24	.41	.27	.37	.55	.30	.43
Cardinality 1	.28	.11	-.52	-.46	-.11	-.76	-.13	-.45
2	.37	.19	.15	.75	.34	1.00	.16	.53
3	.53	.36	-.31	-.37	-.07	-.85	-.33	-.49
4	.58	.51	-.18	-.23	-.11	-.64	-.34	-.54
Comb. Reasoning	.58	.16	.61	.80	.27	1.00	.60	.03
Some-All	-.16	.05	.60	.33	.54	.13	.40	.47
Transitivity L	-.02	.38	-.28	.06	.10	.50	.52	.60
Transitivity W	.02	.11	-1.00	-.68	-.27	-.40	-.14	-.78
Class Inclusion	.07	.41	.35	.64	.50	.77	.31	.47
Cons. id. L	.24	.05	-.20	-.17	-.38	-.66	-.07	-.50
Cons. id. W	-.01	.25	-.27	-.35	-.42	-.78	-.16	-.31
Cons. eq. L	-.26	-.20	.37	.44	.41	.59	.36	.42
Cons. eq. W	-.40	-.10	.41	.13	.37	.71	.33	.42

TABLE 8

RELATIONSHIP OF TRADITIONAL TASKS TO GROUPEMENT PERFORMANCE, PART 2
(GAMMAS, N=120)

	S.C.	Dich.	Comb. R.	S.A.	T.L.	T.W.	I	II	III	IV	V	VI	VII	VIII	C.I.
Serial Corresp.	1.00														
Dich.	.72	1.00													
Comb. Reasoning	.58	.64	1.00												
S.A.	1.00	.64	1.00	1.00											
T.L.	-1.00	-1.00	-.30	-.24	1.00										
T.W.	.44	.29	.07	.20	-.57	1.00									
I	.23	.10	.58	-.16	.02	-.02	1.00								
II	.33	.30	.16	.05	.11	.38	.76	1.00							
III	.28	.38	.61	.60	-1.00	-.28	.69	.57	1.00						
IV	.60	.66	.80	.33	-.68	.06	.70	.27	.52	1.00					
V	.59	.59	.27	.54	-.27	.10	-.00	.54	.55	.72	1.00				
VI	1.00	.76	1.00	.13	-.40	.50	-.49	-.56	.12	.55	.39	1.00			
VII	.49	.38	.60	.40	-.14	.52	-.04	-.07	.06	.31	.56	.60	1.00		
VIII	.66	.44	.03	.47	-.78	.60	.04	.07	.47	.55	.56	.77	.69	1.00	
C.I.	.46	.73	.70	.49	-1.00	.06	.07	.41	.35	.64	.50	.77	.31	.47	1.00

of each groupement task. Means and standard deviations (Tables 9 and 10) were computed by age, sex, and school. These variables were also used in an analysis of variance. As anticipated, grade was highly significant ($p < .001$) for seven of the eight tasks. No significant sex differences were found. Significant school effects were found for groupement II ($p < .0001$), and groupement approached significance (Table 11). Since sex was not significant it was collapsed for further analyses. The school effect for the class inclusion groupement is interesting in that it indicates that specific experience may influence task performance in this area.

PRINCIPLE COMPONENTS ANALYSES (TABLES 12 AND 13)

Examination of the two-factor solutions yielded by the principal components analysis indicates a fairly consistent pattern. The first factor would appear to be congruous with age-related performance. For the kindergarten and third-grade combined subsample all the variables are related to the first factor in the same direction. Groupements I and II have the smallest factor loadings, reflecting the random performance of the younger subjects on these tasks. For the full sample combined, with respect both to gamma and the Pearson "r," all the groupements are strongly related to the first factor.

The second factor appears to be much more task specific and indicates the same relationship among the tasks as does the clustering solution. For the kindergarten and third grade subsample, groupements I, II, and III are highly related, groupements IV and V are intermediate with respect to the second factor, and groupements VI, VII, and VIII are significantly negatively related. For the combined subsample, there is a class and relational split, with all the class items having the same sign and all the relational tasks having the opposite sign. However, in many cases the low factor loadings indicate that certain tasks are again intermediately placed. The gamma solution indicates high scores for groupements I and II. Groupements VI and VII are also highly related in the opposite direction. Factor loadings $\leq .3$ for groupements III, IV, V, and VIII indicate an ambiguity with respect to their position. The solution yielded by the Pearson coefficients relates all the relational groupements to the second factor in one direction, indicates that groupements I and II are highly related in the opposite direction and places groupements III and IV in an intermediary position. Rotation of the matrix of Pearson r's by Varimax procedure forces a class-relation distinction. Rotation of the matrices of both Bentler's coefficients for the kindergarten and third grade subsample and of the gammas for the combined sample are consistent with the clustering result and places groupements I and II at one extreme, with the relational tasks falling at the other extreme.

RELIABILITY OF THE GROUPEMENT TASKS

In order to see how well these tasks discriminate across subjects, coefficient alpha was utilized for the combined sample. The overall alpha of .85 indicates that the tasks discriminate well.

TABLE 9

MEANS BY AGE, SEX, AND SCHOOL

School	Means							
	I	II	III	IV	V	VI	VII	VIII
	Kindergarten Males							
1	3.6	3.8	3.7	3.8	6.4	7.1	5.3	5.3
2	3.5	4.5	3.0	4.0	7.5	7.0	6.5	6.0
3	4.4	5.4	3.5	4.5	6.0	6.0	5.0	4.5
4	4.7	4.7	3.6	3.1	5.9	7.3	6.3	6.1
	Kindergarten Females							
1	4.3	4.2	3.3	3.8	6.3	6.8	5.8	4.8
2	4.0	4.0	3.5	2.0	8.0	7.5	5.0	5.0
3	5.7	6.3	3.0	4.3	6.2	5.1	5.6	5.1
4	4.1	5.0	3.3	3.6	5.9	6.3	4.7	4.7
	Third Grade Males							
1	4.3	4.4	3.4	4.5	7.0	7.9	6.9	6.5
2	4.0	4.6	2.2	5.4	6.4	8.0	6.2	6.2
3	5.3	5.8	3.6	4.6	7.4	7.8	6.1	7.1
4	4.5	4.5	2.8	3.8	6.2	7.7	6.2	5.7
	Third Grade Females							
1	4.1	4.1	4.5	4.8	6.8	7.5	7.0	5.8
2	4.5	4.8	3.1	6.0	7.8	7.9	6.1	5.4
3	5.0	5.4	4.1	4.9	6.7	7.4	5.1	7.0
4	4.6	4.3	3.0	5.3	7.4	8.0	6.0	6.3
	Sixth Grade Males							
1	6.0	6.4	4.6	6.3	7.7	8.0	6.8	7.5
2	5.2	5.4	2.8	5.2	8.0	7.4	7.2	7.0
3	7.1	6.4	4.4	7.3	7.7	8.0	7.3	7.4
4	5.0	5.7	3.2	5.0	7.5	6.5	6.8	6.3
	Sixth Grade Females							
1	6.3	6.3	4.6	7.1	7.5	7.9	6.5	7.0
2	6.1	6.1	5.9	6.9	7.6	8.0	7.0	6.7
3	6.6	7.3	5.6	6.9	6.7	7.4	7.1	7.6
4	6.5	7.0	4.0	4.9	6.5	7.3	6.8	6.4

TABLE 10

STANDARD DEVIATIONS BY AGE, SEX, AND SCHOOL

School	Standard Deviations							
	I	II	III	IV	V	VI	VII	VIII
	Kindergarten Males							
1	1.58	1.87	1.89	1.69	1.65	1.91	2.36	2.11
2	2.12	.71	1.41	2.83	.71	.00	2.12	2.83
3	1.80	1.57	.93	1.75	1.79	1.84	2.28	1.51
4	1.25	1.11	1.40	1.21	1.57	1.11	1.38	1.07
	Kindergarten Females							
1	1.06	1.64	1.87	2.04	1.36	1.60	1.64	1.75
2	.00	.00	.71	.00	.00	.71	1.41	.00
3	1.5	1.94	1.41	1.66	1.99	2.32	2.12	1.76
4	1.35	2.24	2.21	2.51	1.21	1.25	2.92	2.43
	Third Grade Males							
1	.65	.92	2.20	2.34	1.26	.30	2.12	1.29
2	.71	.89	1.79	2.07	2.51	.00	1.92	2.05
3	1.75	1.91	2.56	2.39	1.19	.71	2.64	1.13
4	1.52	1.05	1.60	2.56	.98	.82	1.83	1.37
	Third Grade Females							
1	.35	.83	2.45	1.98	1.49	1.41	1.60	1.75
2	.93	.89	2.53	2.07	.71	.35	2.17	2.62
3	1.53	1.62	3.18	2.34	1.89	1.51	2.34	1.29
4	1.90	.95	2.4	2.36	.79	.00	3.21	1.50
	Sixth Grade Males							
1	1.28	1.31	2.15	2.01	.78	.00	.83	1.45
2	1.10	1.67	2.17	2.77	.00	1.34	1.30	.71
3	.69	1.62	2.51	1.11	.76	.00	1.50	.98
4	1.55	1.63	1.33	1.26	.84	1.97	1.60	1.03
	Sixth Grade Females							
1	1.16	1.39	2.83	1.13	.93	.35	1.69	1.41
2	1.35	1.68	2.12	1.21	1.13	.00	1.00	1.50
3	1.81	1.25	2.15	2.19	1.89	1.51	1.21	.53
4	1.07	.93	2.67	3.09	2.51	1.75	1.83	1.77

TABLE 11

SUMMARY TABLE FOR ANALYSIS OF VARIANCE
(TOTAL SAMPLE, N=180)

Variable	Source of Variation	df	MS	F	p. <
I	School	3	9.45	5.31	.0017
II	School	3	16.09	7.54	.0001*
III	School	3	5.85	1.27	NS
IV	School	3	10.68	2.56	.0571
V	School	3	3.92	1.88	NS
VI	School	3	4.46	2.74	.0456
VII	School	3	1.41	.36	NS
VIII	School	3	2.06	.80	NS
I	Grade	2	61.54	34.60	.0001*
II	Grade	2	56.21	26.34	.0001*
III	Grade	2	24.01	5.22	.0064
IV	Grade	2	83.77	20.07	.0001*
V	Grade	2	17.32	8.31	.0004*
VI	Grade	2	26.35	16.15	.0001*
VII	Grade	2	29.77	7.64	.0007*
VIII	Grade	2	60.88	23.55	.0001*
I	Sex	1	5.27	2.96	NS
II	Sex	1	3.54	1.66	NS
III	Sex	1	9.66	2.10	NS
IV	Sex	1	4.49	1.08	NS
V	Sex	1	.39	.19	NS
VI	Sex	1	2.19	1.34	NS
VII	Sex	1	1.56	.40	NS
VIII	Sex	1	3.78	1.46	NS
I	Sex x School	3	.46	.26	NS
II	Sex x School	3	.79	.37	NS
III	Sex x School	3	3.06	.67	NS
IV	Sex x School	3	1.07	.26	NS
V	Sex x School	3	1.48	.71	NS
VI	Sex x School	3	1.08	.66	NS
VII	Sex x School	3	1.14	.29	NS
VIII	Sex x School	3	1.97	.76	NS
I	Grade x School	6	.58	.33	NS
II	Grade x School	6	2.12	1.00	NS
III	Grade x School	6	2.60	.56	NS
IV	Grade x School	6	5.05	1.21	NS
V	Grade x School	6	1.19	.57	NS
VI	Grade x School	6	3.61	2.21	.04
VII	Grade x School	6	2.43	.62	NS
VIII	Grade x School	6	3.85	1.49	NS

* significance level
p < .001

(continued)

TABLE 11

(continued)

Variable	Source of Variation	df	MS	F	p. <
I	Grade x Sex	2	1.57	.88	NS
II	Grade x Sex	2	2.63	1.23	NS
III	Grade x Sex	2	6.17	1.34	NS
IV	Grade x Sex	2	1.50	.36	NS
V	Grade x Sex	2	3.64	1.75	NS
VI	Grade x Sex	2	1.55	.95	NS
VII	Grade x Sex	2	.08	.02	NS
VIII	Grade x Sex	2	.27	.10	NS
I	Grade x Sex x School	6	2.55	1.43	NS
II	Grade x Sex x School	6	.89	.42	NS
III	Grade x Sex x School	6	1.77	.38	NS
IV	Grade x Sex x School	6	2.34	.56	NS
V	Grade x Sex x School	6	1.58	.76	NS
VI	Grade x Sex x School	6	.74	.45	NS
VII	Grade x Sex x School	6	2.20	.57	NS
VIII	Grade x Sex x School	6	1.53	.59	NS

TABLE 12

PRINCIPLE COMPONENTS ANALYSIS OF THE KINDERGARTEN AND THIRD GRADE SUBSAMPLE (N=120)

Bentler's Coefficient
Unrotated Solution

Var/Factor	1	2
I	-.389	-.773
II	-.236	-.702
III	-.578	-.408
IV	-.724	-.130
V	-.649	.144
VI	-.482	.646
VII	-.471	.507
VIII	-.730	.209

Bentler's Coefficient
Principle Components (N=120)

Var/Factor	1	2
I	-.059	.863
II	-.154	.725
III	.290	.646
IV	.557	.480
V	.632	.206
VI	.744	-.311
VII	.664	-.197
VIII	.734	.192

Factor	Factor Var.	% Total Factor Var.	% Total Var.
1	2.3535	52.5	29.4
2	2.1324	47.5	26.7
			56.1

TABLE 13

PRINCIPLE COMPONENTS ANALYSIS OF THE FULL SAMPLE (N=180)

Pearson Product Moment Correlation Unrotated Solution			Gamma Unrotated Solution		
Var/Factor	1	2	Var/Factor	1	2
I	-.621	-.480	I	-.711	.702
II	-.533	-.636	II	-.553	.757
III	-.556	-.249	III	-.842	.206
IV	-.624	-.154	IV	-.916	.092
V	-.452	.412	V	-.715	-.314
VI	-.473	.359	VI	-.616	-.728
VII	-.435	.497	VII	-.616	-.488
VIII	-.533	.573	VIII	-.837	-.241

Pearson Product Moment Correlations Rotated Solution (N=180)

Var/Factor	1	2
I	.784	.029
II	.817	-.146
III	.586	.165
IV	.578	.282
V	.083	.606
VI	.133	.579
VII	.015	.761
VIII	.034	.774

Factor	Factor Var.	% Total Factor Var.	% Total Var.
1	1.9857	51.5	24.8
2	1.8668	48.5	23.3
			48.1

Gammas Rotated Solution

Var/Factor	1	2
I	.077	.996
II	-.079	.935
III	.500	.708
IV	.631	.670
V	.746	.231
VI	.942	-.146
VII	.785	.035
VIII	.790	.367

Factor	Factor Var.	% Total Factor Var.	% Total Var.
1	3.3453	52.5	41.8
2	3.0250	47.5	37.8
			79.6

DIMENSIONALITY

Brainerd's (1972b) initial premise concerning the groupement is that it is a two-dimensional construct. In order to attempt to replicate this finding, primary emphasis was placed on techniques designed to search for the existence of underlying "factors," such as multidimensional scaling, clustering, and factor analysis. Initially the data were handled dichotomously. Using pass-fail scores derived for each of the eight groupement tasks, two x two contingency tables were constructed for all pairs of tasks and both gammas and Bentler's coefficients of monotonicity were computed.

Initial dimensional analysis was made utilizing the dichotomous data and subsequently the intervally scored data. Since the nonparametric procedures, i.e., multidimensional scaling and clustering, only utilize the ranks of the correlation coefficient no differences were found between the solutions found with the gammas and Bentler's coefficients, both derived from the same contingency tables. This is a necessity since for the two x two case $\gamma = \frac{ad - bc}{ad + bc}$ and $m = \frac{ad - bc}{ad + bc + 2\sqrt{abcd}}$. Although the range of gamma values is wider, the rank order and signs remain invariant. The solutions are therefore identical. (See Appendix B)

MULTIDIMENSIONAL SCALING

For the collapsed grade subject sample, solutions were considered in one, two, and three dimensions. Scaled in one dimension the tasks were ordered as follows: II, I, III, IV, V, VIII, VI, VII. Although this solution separates the "class" and "relational" groupement tasks, the poor stress (.21) indicates that this is not a very satisfactory solution.

In two dimensions (see Figure 1), a much more satisfactory stress of .09 was found. As indicated in Figure 1, groupements I, II, and III are placed at one extreme of the picture and groupements VI and VII are at the opposite end. Groupement VIII is also strongly related to groupements VI and VII. This distinction corresponds to the relative ease and difficulty of performance indicated by the percentages passing the tasks. An ambiquity is indicated for groupements IV and V and the clear distinction between class and relational groupements is not found. The secondary split grouping tasks I, II, VII, VIII and III, IV, V, VI completely crosses the class and relational distinction and is the one that needs to be explored. As indicated in Figure 1, groupements I and II are located in the same quadrant and are almost identical in the first dimension. Since the task format and stimulus are identical this is not surprising. Groupements VII and VIII also are conceptually and task-specifically related very closely and are also in the same quadrant. The second dimension crosses these distinctions ordering the tasks from the strongest II, VII, VIII, I. In the lower half of the picture the strongest tasks in the second dimension are groupements III and V, with IV and VI appearing very similar.

In three dimensions, with a stress of .02, the first and second dimensions remain the same as in two dimensions. Added is a third dimension that rank orders the tasks as follows: IV, VI, VIII, I, II, III, V, VII, with V and VII being the only positive values. This seems to add relatively little new information to the second dimension, other than adding an asymmetric operation for groupements V and VII. Although stress is lowered, this is only to be expected due to the small number of tasks.

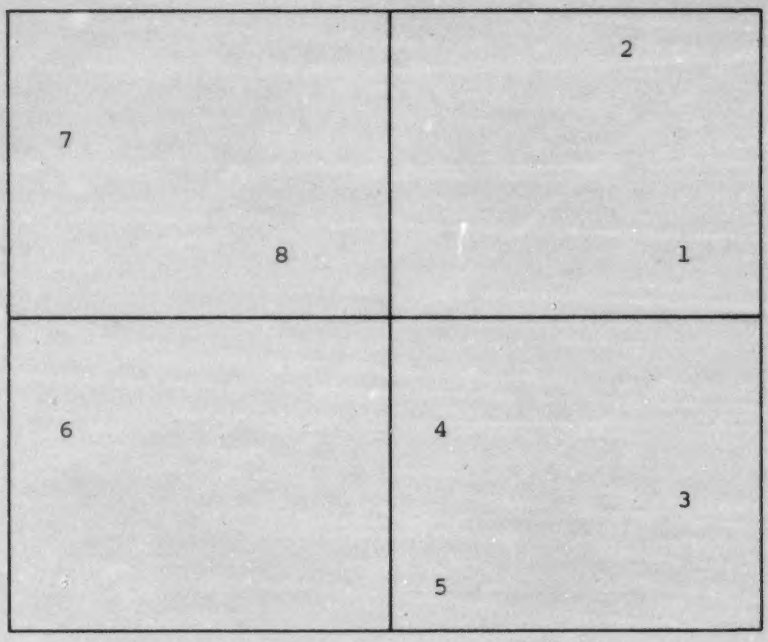

Figure 1. Two dimensional representation. (N=180).

For a combined subject sample of kindergarten and third grade subjects, the first dimension, with stress = .16, makes the class and relational split ordering tasks as follows: II, I, III, IV, V, VIII, VII, VI, indicating the strong influence of the random performance of the younger subjects on the more difficult tasks. A slight reduction in stress (.13) for the two dimensional configuration (see Figure 2) retains the class and relational distinction, although placing groupements IV and V close together and makes a second distinction which seems to separate groupement I, II, VII, VIII from III, IV, V, VI as was shown in the examination of the performance of the entire subject sample and may indicate (by comparing the stress values) that this dimension becomes more influential with age and associated mastery of the more difficult groupement. Again three dimensions reduces the stress (.05) but adds uninterpretable information (see Figure 3).

CLUSTERING

Gammas were chosen to attempt to control difficulty levels between tasks. Clustering was performed with the gammas in order to compare its solution with the scaling solutions and with Pearson product moment correlations in order to see if the results would differ.

With the gammas the clustering solution found for the combined kindergarten and third grade sample indicates three major clusters (Figure 4): (I, II, III), (IV, V), (VI, VII, VIII). The clustering solution for the across-grade sample, which does not indicate a strong relationship, indicates the following three clusters (Figure 5): (I, II), (III, IV, V), (VI, VII, VIII). Across grades, clustering the r's indicates a strong distinction between class and relations, finding two strong clusters (Figure 5): (I, II, III, IV) and (V, VI, VII, VIII). This may indicate either that performance factors play a large part in the difficulty distinction or that interval data reveals more subtle differences. At any rate it indicates that the choice of correlational measures is crucial.

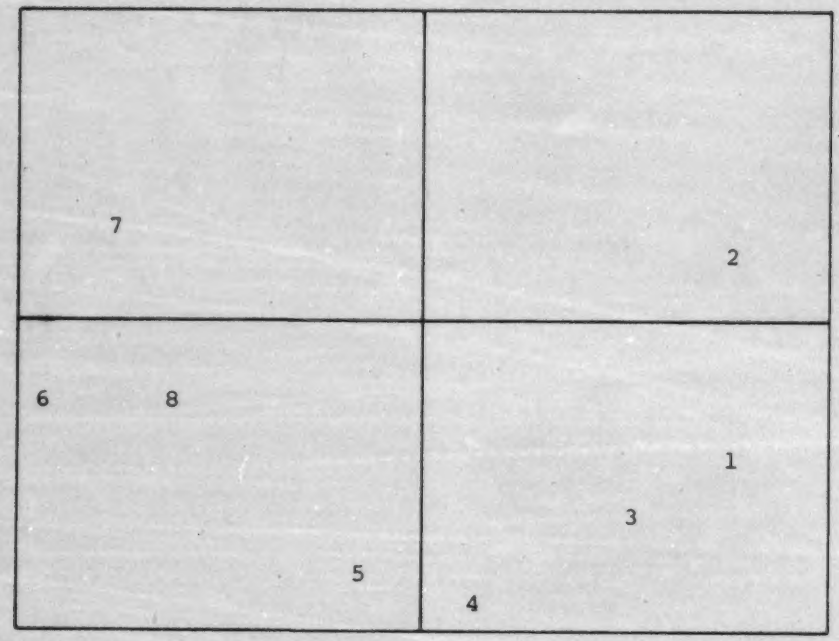

Figure 2. Two dimensional representation. (N=120).

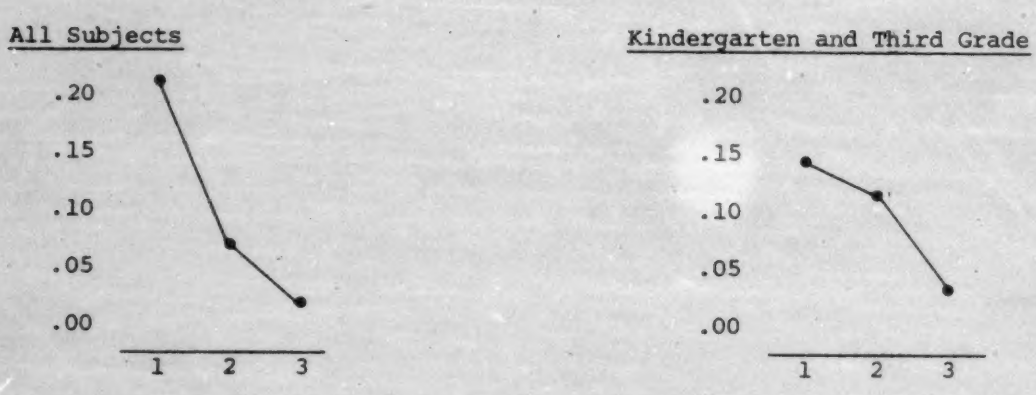

Figure 3. Multidimensional stress curves.

Bentler's m and gamma

	{I} {II} {III} {IV} {V} {VI} {VII} {VIII}	
	{VI,VIII} {I} {II} {III} {IV} {V} {VII}	.83
	{VI,VIII} {I,II} {III} {IV} {V} {VII}	1.0
	{VI,VIII} {I,II} {III} {IV,V} {VII}	1.0
	{VI,VII,VIII} {I,II} {III} {IV,V}	1.0
*	{VI,VII,VIII} {I,II,III} {IV,V}	.947
	{IV,V,VI,VII,VIII} {I,II,III}	.94
	{I,II,III,IV,V,VI,VII,VIII}	.79

Figure 4. Clustering of Bentler's m and gamma N=120 (K,3)

Interval Data

	{I} {II} {III} {IV} {V} {VI} {VII} {VIII}
	{I,II} {III} {IV} {V} {VI} {VII} {VIII}
	{I,II} {III} {IV} {V} {VI} {VII,VIII}
	{I,II} {III,IV} {V} {VI} {VII,VIII}
	{I,II,III,IV} {V} {VI} {VII,VIII}
	{I,II,III,IV} {V,VII,VIII} {VI}
*	{I,II,III,IV} {V,VI,VII,VIII}
	{I,II,III,IV,V,VI,VII,VIII}

Gammas (K,3,6)

	{I} {II} {III} {IV} {V} {VI} {VII} {VIII}	
	{I,II} {III} {IV} {V} {VI} {VII} {VIII}	.59
	{I,II} {III} {IV} {V} {VI,VIII} {VII}	1.00
	{I,II} {III,V} {IV} {VI,VIII} {VII}	.92
	{I,II} {III,V} {IV} {VI,VII,VIII}	.86
*	{I,II} {III,IV,V} {VI,VII,VIII}	.78
	{I,II} {III,IV,V,VI,VII,VIII}	.75
	{I,II,III,IV,V,VI,VII,VIII}	.497

Figure 5. Clustering N=180 (K,3,6)

* Optimal level

COMPOSITION-REVERSE OPERATIONS

Considering the cross-sectional percentage patterns, the composition operation of the class groupement appears consistently more difficult than its inverse. This is most extreme in the case of groupements I and II, with a narrowing of the gap seen between third and sixth grade. The difference between the composition-reciprocal operation is not nearly so pronounced and the narrowing of the ratio with age is not as drastic (see Table 3).

Collapsing across age in order to investigate if operation is a better predictor of success than task, correlations within task were compared with correlations between tasks and within operation (Table 14). The correlations were also used as input in the SSA program. No support for a consistent pattern of specific composition or reverse ability was indicated.

These findings illustrate discrepancies that may be found by considering age groups separately when looking for developmental patterns. It is also misleading to look at percentage data, since it is impossible to discern how one individual performs on each task and whether mastery of one task in any way facilitates mastery of another task.

TRADITIONAL PIAGETIAN TASKS

Two tasks (combinatorial reasoning and class inclusion) from a traditional Piagetian task battery were administered to the entire subject sample and are considered to be related to the logical groupements. Since, combinatorial reasoning may also be considered a formal task, it may be considered the most difficult task of the array. Rank ordering the correlations of these tasks with the groupements indicate the following:

Class Inclusion		Combinatorial Reasoning	
IV	.82	VI	1.00
VI	.80	IV	.79
VIII	.59	I	.63
I	.58	III	.60
II	.56	II	.58
III	.53	VII	.49
VII	.39	V	.48
V	.31	VIII	.41

It is interesting to note that although the class inclusion task is considered an operationalization of groupements I and II, it is most highly related to relational tasks. Therefore the class inclusion task and groupements I and II cannot truly be considered alternate measures of the same ability.

TABLE 14

COMPOSITION-INVERSION RELATIONSHIP
OF CLASS GROUPEMENT N=180

		I		II		III		IV	
		C	I	C	I	C	I	C	I
I	C		.54	.99	.43	.86	.62	.85	.43
	I			.73	.92	.56	.79	.45	.71
II	C				1.00	.81	.56	.75	.44
	I					.46	.72	.80	.73
III	C						.80	.76	.87
	I								.75
IV	C								.76
	I								

COMPOSITION-RECIPROCAL RELATIONSHIP
OF RELATIONS GROUPEMENT (N=180)

		V		VI		VII		VIII	
		C	R	C	R	C	R	C	R
V	C		.86	.54	.48	.22	.40	.39	.53
	R			.68	.69	.56	.79	.54	.69
VI	C				.96	.64	.59	.66	.75
	R					.72	.63	.81	.70
VII	C						.80	.62	.49
	R							.70	.81
VIII	C								.82
	R								

		V		VI		VII		VIII	
		C	R	C	R	C	R	C	R
I	C	.32	.64	.25	.13	-.00	.60	.63	.66
	R	.28	.62	.79	.79	.62	.56	.57	.75
II	C	.30	.43	.05	.03	.11	.39	.45	.62
	R	.47	.57	.77	.52	.58	.47	.55	.65
III	C	.62	.38	.73	1.00	-.06	.49	.65	.90
	R	.49	.60	.64	.76	.32	.54	.42	.46
IV	C	.64	.45	.66	.51	.10	.37	.48	.52
	R	.46	.73	.82	.88	.49	.50	.38	.62

VI

DISCUSSION

INITIAL CONSIDERATIONS

This study was concerned with whether a "class-relations" split could be substantiated. Results did not indicate, across age groups, that this was a valid assumption. At an early age (kindergarten and third grade) a class-relations distinction was found; however, it was a very weak distinction, with groupements IV and V being extremely similar.

The uniqueness of groupements I and II raises theoretical questions concerning either the level at which class union is achieved or the language with which it was expressed in this study. Neimark and Slotnick (1970) found that the language of inclusion is extremely late in appearing. They postulated that class union should be considered a formal, rather than a concrete, ability. They substantiated this reasoning by postulating that it is this process that is responsible for such formal level tasks as combinatorial reasoning.

The correlational data, with respect to both the kindergarten and third grade subjects and to the full sample, indicates that the groupements, as it was measured here, does not completely account for mastery of other concrete level tasks. Extremely poor predictive association was found for the transitivity and conservation tasks, as well as for the number tasks.

One may speculate in the case of the groupements whether there is any practical reason for assessing each structure independently, or whether the abilities of the middle childhood period are tied to several structures in such a way that the child will never be required to independently utilize them. Testing the correspondence between logical and psychological structures itself is difficult, requiring close observation of individuals to see if there is "contemporaneous emergence of formally similar structures and close relationship of these structures throughout development in middle childhood [Shantz, 1967, p. 241]." This indicates that a developmental pattern is one that is invariant and is the same across subjects. Since cross-sectional assessment does not provide for comparisons of developmental patterns across subjects, it is inadequate in this respect.

DEVELOPMENTAL SYNCHRONY

The task split, as indicated by the scaling procedure and the factor analytic procedure, indicates that addition of the second dimension is extremely necessary to account for a large percentage of the total variance. It is this second dimension that combines two class and two relational tasks. The third dimension almost completely negates the stress value in the scaling solution, indicating the most complete representation of these tasks. Although it is true that as the number of tasks increases, stress is always reduced; however, this would not account for such a drastic reduction. Therefore, it may be concluded that similarities between class and relational operations contribute in a very meaningful way to

the whole picture. This similarity becomes increasingly integrated with age, as groupements I and II are passed with a probability greater than change. An ordering according to difficulty no longer provides even a poor fit (stress = .21). This would tend more to support synchrony than disprove it. Performance considerations cause a split between class and relational tasks, most notable in the younger samples. This split becomes much more distinct, even with the entire subject sample, when difficulty level is uncontrolled (as with use of the Pearson r) and when the data is intervally scored, indicating that either (1) task-specific difficulty level contributes spuriously to task distinctions, or (2) interval data allows comparison between subjects answering only a few questions correctly and that this minimal performance may indicate a different picture, confounding the issue of task mastery with guessing or transitionary behavior, or (3) there is a valid split masked by methodological problems.

METHODOLOGICAL PROCEDURES

Although the techniques used in this study are primarily descriptive, the lack of exact statistical procedures for dealing with developmental issues suggests a future for them in psychology. The solutions found using the multidimensional scaling, clustering, and factor analytic procedures indicated consistent solutions. The nonparametric techniques have the additional advantages of not making strict assumptions and requiring only rank-order data for input. The pictorial representation of the scaling solution aided interpretation.

Since the goal of much Piagetian research is to look for underlying abilities, these descriptive techniques seem more appropriate than many procedures previously utilized, i.e., correlational, analysis of variance, percentages.

SUMMARY AND CONCLUSIONS

The present study tends to support evidence for a unitary, age-related property within the groupement which is manifested during the middle childhood period. Although task specific relationships are found, they do not seem to support a distinct "class-relation" split. Indications are, however, that the underlying model may not be an additive model but an interactive one, in which some tasks, most notably class-inclusion ones (groupement I and II), show greater variances across the age-span. This may indicate that task specific properties cause fluctuation or that experiential forces have a differential influence. The latter position is supported by the ANOVA results which indicate a significant school effect for class inclusion.

The extreme difficulty level of the class inclusion tasks as reported here and discussed in detail in previous studies (Brainerd and Kaszor, 1974; Lagattuta, 1970; Hooper, et al., 1974) indicates that inclusion of these tasks as concrete level abilities, while logically supportable, may in actuality be questionable. However, viewed from the position that, in reality, some task performances may develop at a consistent rate while others show rapid growth spurts, this separation of inclusion tasks may be due to methodological considerations, especially the choice of measurement points. Data from measurement confined to a limited age-range may be misleading and, thus, a crucial consideration since manipulation of measurement time may indicate support for contradictory hypotheses.

Indications are that traditional concrete level performance is not completely subsumed by individually operationalizing each of the hypothesized groupements. This may suggest a complementary model of class and relational abilities, which is not new to Piagetian theory with respect to conservation and number concepts (Piaget, 1952). However, methodological and measurement considerations make Piagetian predictions difficult to test in a strict statistical fashion for which they were not intended. It is evident that the theoretical position of developmental synchrony may not be discarded on the basis of these findings.

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APPENDIX A

ASSESSMENT BATTERY

Successive Comparisons

Task I

E presents 4 sticks (3,4,5,6) in scrambled fashion and not in a straight line.

HERE ARE SOME STICKS, I WANT YOU TO ARRANGE THEM IN ORDER.

If S hesitates E asks,

FIND THE SHORTEST STICK AND PLACE IT HERE (to the S's right) pause AND FIND THE LONGEST STICK AND PLACE IT HERE (to S's left). NOW PUT ALL THE OTHER STICKS IN ORDER BETWEEN THE LONGEST AND THE SHORTEST.

E completes task if necessary and asks;

WHY DID YOU (WE) ORDER THE STICKS THAT WAY?

E tries to get S to use the increasing length of the sticks as the criterion for the arrangement.

Successive Comparison

Task II

E presents sticks (1,3,4,6,7,8,10) in a scrambled fashion and not in a straight line.

HERE WE HAVE SOME STICKS. I WANT YOU TO ARRANGE THESE STICKS IN ORDER. REMEMBER ALL THE STICKS HAVE TO BE IN ORDER.

If the child hesitates E asks;

FIND THE SHORTEST STICK AND PLACE IT HERE (to the S's right) pause AND FIND THE LONGEST STICK AND PLACE IT HERE (to the S's left). NOW PLACE ALL THE OTHER STICKS IN ORDER BETWEEN THE LONGEST AND THE SHORTEST.

E helps S to finish the task if necessary.

ARE YOU FINISHED? ARE THEY JUST RIGHT?

E arranges sticks in correct order before proceeding to next task.

Additive Seriation

E adjusts the array so that there is an inch space between the sticks and a two inch space between the sticks where a stick is to be added. Pause.

E places the remaining sticks (2,5,9) an inch apart between the original array and the S in order and in the same ascending direction as the original array.

HERE ARE THREE MORE STICKS THAT GO WITH THE OTHER STICKS. CAN YOU SHOW ME WITHOUT TOUCHING THE STICKS WHERE THIS STICK (pointing to 2) WOULD GO INTO THE ORDER OF STICKS? E repeats this procedure with stick 5, then 9.

Prediction:

NOW PUT THESE STICKS INTO THE ORDER WITH THE OTHER STICKS. PUT THEM IN WHERE THEY BELONG.

If S fails to understand the task, E places one of the sticks for him.

After S has finished E asks;

HAVE YOU PLACED THE STICKS THE WAY YOU WANT THEM? CHECK AND MAKE SURE.

Placement: (If E helps, score that stick incorrect.)

Serial Correspondence

E places the circles between the S and the order array of sticks in a mixed fashion.

HERE ARE SOME CIRCLES, I WANT YOU TO ARRANGE THESE CIRCLES IN ORDER. REMEMBER ALL OF THE CIRCLES HAVE TO BE IN ORDER. PUT THEM IN THE SAME WAY AS THE STICKS.

If the child hesitates E asks;

FIND THE SMALLEST CIRCLE AND PLACE IT HERE (to the side of the shortest stick) pause AND FIND THE LARGEST CIRCLE AND PLACE IT HERE (to the side of the largest stick). NOW PLACE ALL THE OTHER CIRCLES IN ORDER BETWEEN THE LARGEST AND THE SMALLEST CIRCLE.

The circles should be very close together but not touching.

NOW LET'S MATCH EACH STICK TO THE CIRCLE IT GOES WITH. Pause to see if S can make the correct correspondence order.

If he hesitates E asks:

PUT THE LONGEST STICK WITH THE LARGEST CIRCLE. Pause. PUT THE SHORTEST STICK WITH THE SMALLEST CIRCLE. NOW PUT EACH OF THESE STICKS WITH THE CIRCLE THAT IT GOES WITH. THEY MUST BE THE RIGHT SIZE FOR EACH OTHER.

If S responds incorrectly E replaces the sticks and circles to correspondence and explains;

EACH STICK GOES WITH A CIRCLE. THEY ARE THE RIGHT SIZE FOR EACH OTHER.

E extends the stick array so there is an extra two sticks at each end of the circle array. Place sticks so the longest sticks are roughly 4 inches above the largest circles. Order sticks over spaces between circles. E points to stick 5 and asks:

POINT TO THE CIRCLE THAT GOES WITH THIS STICK. THEY HAVE TO BE THE RIGHT SIZE FOR EACH OTHER.

E repeats this procedure for sticks 2 and 7.

Serial Correspondence continued

S may not move the sticks or circles.

E returns the sticks and circles to correspondence and once again establishes the relationship between the circles and sticks.

E compresses sticks (4 inches above largest circle) so all the sticks are one inch apart and clustered between circles 4 and 8. E points to stick 6.

POINT TO THE CIRCLE THAT GOES WITH THIS STICK. THEY HAVE TO BE THE RIGHT SIZE FOR EACH OTHER.

E repeats this procedure for stick 8, then 3.

E returns the sticks and circles to correspondence and once again establishes the relationship between the circles and sticks.

E scrambles the sticks. E points to stick 4 and asks:

WHAT CIRCLE GOES WITH THIS STICK? YOU CAN DO ANYTHING YOU WANT WITH THE STICKS. (Don't tell to construct the order even thought it is permissible.) REMEMBER, THEY HAVE TO BE THE RIGHT SIZE FOR EACH OTHER.

E repeats this procedure for stick 9, then 5. If S moves the sticks during a trial, E scrambles the sticks again.

If S does not reconstruct - then E asks:

HOW CAN YOU BE SURE THAT THIS STICK (5) GOES WITH THIS CIRCLE (5)? YOU CAN DO ANYTHING YOU WANT WITH THE STICKS.

reconstructs
doesn't know or guessed
irrelevant justification
relevant justification but does not reconstruct

Dichotomies

DIVIDE ALL THESE DRAWINGS INTO TWO BUNCHES. PUT ONE KIND HERE AND ONE KIND HERE. For 2nd and 3rd Dichotomies, E adds, BUT DO IT IN A DIFFERENT WAY THAN BEFORE.

1st Dichotomy

a. Exhaustive sort with number as only criterion
b. Exhaustive sort with shape as only criterion
c. Exhaustive sort with color as only criterion
d. Other

2nd Dichotomy

a. Exhaustive sort with number as only criterion
b. Exhaustive sort with shape as only criterion
c. Exhaustive sort with color as only criterion
d. Other

3rd Dichotomy

a. Exhaustive sort with number as only criterion
b. Exhaustive sort with shape as only criterion
c. Exhaustive sort with color as only criterion
d. Other

Conservation of Number

Materials:

20 plastic chips

Procedure: The experimenter and subject construct two parallel rows of evenly spaced chips in the center of the table. There is a precise perceptual correspondence between the elements of the two rows.

1.) Prediction: Leaving the rows exactly as they are, the experimenter asks the following questions:

   (a) IF I WERE TO PUSH THE CHIPS IN THIS ROW (pointing to the row nearest the experimenter) VERY CLOSE TOGETHER, WOULD THE TWO ROWS STILL HAVE THE SAME NUMBER OF CHIPS?

   HOW DO YOU KNOW?

   (b) IF I WERE TO PUSH THE CHIPS IN THIS ROW (indicating the same row) VERY CLOSE TOGETHER, WOULD ONE OF THE ROWS HAVE MORE CHIPS?

   (c) IF I WERE TO PUSH THE CHIPS IN THIS ROW (indicating the same row) VERY CLOSE TOGETHER, WOULD ONE OF THE ROWS HAVE FEWER CHIPS?

2.) Deformation: The experimenter pushes the chips in the nearest row together until they touch. The row nearest the subject is now roughly three times as long as the other row. The experimenter asks the following (randomly ordered) questions:

   (a) DO THESE TWO ROWS HAVE THE SAME NUMBER OF CHIPS?

   (b) DOES ONE OF THE ROWS HAVE MORE CHIPS NOW?

   (c) DOES ONE OF THE ROWS HAVE FEWER CHIPS NOW? HOW DO YOU KNOW?

3.) E replaces rows to original order and then removes one chip (2nd from either end) from the row nearest E.

   (a) DO THESE ROWS HAVE THE SAME NUMBER OF CHIPS?

   (b) HOW DO YOU KNOW?

Cardinality I

NOW WE ARE GOING TO PLAY A GAME. I AM GOING TO SHOW YOU SOME CARDS WITH TWO ROWS OF DOTS ON THEM. I WANT YOU TO FIGURE OUT WHETHER THERE ARE THE SAME NUMBER OF RED DOTS AS GREEN DOTS, OR IF ONE OF THE TWO ROWS HAS MORE DOTS.

ONE SPECIAL RULE YOU MUST FOLLOW - YOU CANNOT COUNT THE DOTS - YOU HAVE TO FIGURE OUT THE ANSWER SOME OTHER WAY.

One at a time E presents a card on the table in front of S with the green dots closest to S. S may not touch the card. E asks the questions below for each card. Allow as much time as needed.

Cardinality A (Red = 8, Green = 6)

ARE THERE THE SAME NUMBER OF RED DOTS AS GREEN DOTS ON THIS CARD?

No* Yes

DOES ONE OF THE ROWS HAVE MORE DOTS?

Yes* No

If yes, WHICH ONE? Red* Green
If no,

HERE IS A NEW CARD.

Cardinality B (Red = 8, Green = 6)

ARE THERE THE SAME NUMBER OF RED DOTS AS GREEN DOTS ON THIS CARD?

No* Yes

DOES ONE OF THE ROWS HAVE FEWER DOTS?

Yes* No

If yes, WHICH ONE? Green* Red
If no,

HERE IS A NEW CARD.

Cardinality C (Red = 8, Green = 10)

ARE THERE THE SAME NUMBER OF RED DOTS AS GREEN DOTS ON THIS CARD?

No* Yes

DOES ONE OF THE ROWS HAVE MORE DOTS?

Yes* No

If yes, WHICH ONE? Green* Red
If no,

* correct response

Cardinality I continued

HERE IS A NEW CARD.

Cardinality D (Red = 8, Green = 10)

ARE THERE THE SAME NUMBER OF RED DOTS AS GREEN DOTS ON THIS CARD?

No * Yes

DOES ONE OF THE ROWS HAVE FEWER DOTS?

Yes* No

If yes, WHICH ONE? Red* Green
If no,

HERE IS A NEW CARD.

Cardinality E (Red = 8, Green = 8)

ARE THERE THE SAME NUMBER OF RED DOTS AS GREEN DOTS ON THIS CARD?

Yes* No

DOES ONE OF THE ROWS HAVE MORE DOTS?

No * Yes

HERE IS A NEW CARD.

Cardinality F (Red = 8, Green = 8)

ARE THERE THE SAME NUMBER OF RED DOTS AS GREEN DOTS ON THIS CARD?

Yes* No

DOES ONE OF THE ROWS HAVE FEWER DOTS?

No * Yes

To check against counting watch for lip movements, and rhythmic motor responses.

E's judgment of counting on the part of S.

No * Yes

E asks S, DID YOU FIND THAT YOU COULD ANSWER ALL THE QUESTIONS WITHOUT COUNTING THE DOTS?

Yes* No

DID YOU HAVE TO COUNT SOMETIMES?

No * Yes

* correct reponse

Cardinality I continued

CAN YOU EXPLAIN HOW YOU WERE ABLE TO ANSWER THE QUESTIONS?

matched circle (correspondence)
doesn't know
irrelevant justification
counted circles
some were bunched together and
others were spread apart

Cardinality II

Story A SUPPOSE THAT I HAD 8 COOKIES AND YOU HAD 6 COOKIES.

WOULD WE BOTH HAVE THE SAME NUMBER OF COOKIES?

No* Yes

WOULD ONE OF US HAVE MORE COOKIES?

Yes* No

If yes, WHICH ONE? E * S
If no,

Story B SUPPOSE THAT I HAD 4 COOKIES AND YOU HAD 4 COOKIES.

WOULD WE BOTH HAVE THE SAME NUMBER OF COOKIES?

Yes* No

WOULD ONE OF US HAVE FEWER COOKIES?

No* Yes

E presents Card A_1 and asks:

HOW MANY DOTS ARE ON THIS CARD?

8* Other

If other, E helps S count the dots.

E places A & B between S and Card A_1 and asks:

HOW MANY DOTS ARE ON THIS CARD?

8* Other

If other, E helps S count the dots.

ARE THERE THE SAME NUMBER OF DOTS ON THESE CARDS?

Yes* No

DOES ONE OF THE CARDS HAVE FEWER DOTS?

No* Yes

E presents Card B_1 and asks:

HOW MANY DOTS ARE ON THIS CARD?

6* Other

If other, E helps S count the dots.

* correct response

Cardinality II continued

E places Card A & B between S and Card B_1 and asks:

HOW MANY DOTS ARE ON THIS CARD?

8* Other

If other, E helps S count the dots.

ARE THERE THE SAME NUMBER OF DOTS ON THESE CARDS?

No* Yes

**DOES ONE OF THE CARDS HAVE MORE DOTS?

Yes* No

**If yes, WHICH ONE? Green* Red

If no,

**E does not score these questions until both are asked.

*correct response

Some-All

1. LOOK AT ALL THE RED FIGURES, ARE ALL THE RED FIGURES TRIANGLES?

   Yes*, No, Other

2. LOOK AT ALL THE TRIANGLES, ARE ALL THE TRIANGLES RED?

   No*, Yes, Other

3. LOOK AT ALL THE CIRCLES, ARE ALL THE CIRCLES BLUE?

   Yes*, No, Other

4. LOOK AT ALL BLUE FIGURES, ARE ALL THE BLUE FIGURES CIRCLES?

   No*, Yes, Other

* correct response

Combinatorial Reasoning

SHOW ME ALL THE PAIRS YOU CAN MAKE USING THESE COLORS. THE TWO RULES ARE THAT EACH PAIR MUST HAVE TWO COLORS AND EACH TIME YOU ARE TO PUT DOWN A NEW OR DIFFERENT PAIR OF COLORS.

A. Four Colors

Pairs Formed: R (G,Y,B)
G (Y,B)
Y (B)

Number of correct pairs. _____

Number of repeated pairs. _____

Approach:

1. X + remaining colors, Y + remaining colors, etc.
2. X +3, Y + 3, etc.
3. Random, no apparent system
4. Begins one of the above, and shifts to another of the above.
5. Other

B. Six Colors

Pairs Formed: R (G,Y,B,W,O)
G (Y,B,W,O)
Y (B,W,O)
B (W,O)
W (O)

Number of correct pairs. _____

Number of repeated pairs. _____

Approach:

Same as above.

C. Eight Colors*

Pairs Formed: Br (R,G,Y,B,W,O,Lb)
Lb (R,G,Y,B,W,O)

Number of correct pairs. _____

Number of repeated pairs. _____

Combinatorial Reasoning continued

Approach:

Same as above.

* Administer only if 12 of the derived 15 possible correct pairs (i.e., correct pairs minus repeated pairs) were formed with six colors.

Transitivity of Length

Materials:

27-cm blue stick
28-cm blue stick
28-cm white stick

Instructions:

The E places the board, having a 27-cm blue stick and 28-cm blue stick glued down approximately one arm's length apart, 8-10 inches from the S in the middle of the table. The sticks are positioned such that the midpoint of each stick is in direct relation to the other stick. Taking the 28-cm white stick and placing it in the middle of the board between the two blue sticks, the E says:

HERE ARE SOME STICKS WE WILL BE WORKING WITH.

The E then places the 28-cm white stick next to the 28-cm blue stick, making the ends nearest the S even with one another, and so the S can observe the sticks to be of equal length. The S is required to verbalize this latter fact.

ARE THESE TWO STICKS THE SAME LENGTH?

Yes ___ No ___ I Don't Know ___ No Response ___

Next, the E places the 28-cm white stick next to the 27-cm blue stick, again making the ends nearest the S even with one another, and so the S can observe that the white stick is the longer of the two. The S is required to verbalize this latter fact.

IS ONE OF THE STICKS LONGER?

Yes ___ No ___ I Don't Know ___ No Response ___

(If "Yes," then) WHICH ONE?

White ___ Blue ___ I Don't Know ___ No Response ___

Transitivity of Length continued

Finally, the E removes the white stick from the table, and asks the following questions:

(a) ARE THESE TWO STICKS THE SAME LENGTH?

Yes ___ No ___ I Don't Know ___ No Response ___

(b) IS ONE OF THE STICKS LONGER?

Yes ___ No ___ I Don't Know ___ No Response ___

(If "Yes," then) WHICH ONE?

28-cm ___ 27-cm ___ I Don't Know ___ No Response ___

(c) IS ONE OF THE STICKS SHORTER?

Yes ___ No ___ I Don't Know ___ No Response ___

(If "Yes," then) WHICH ONE?

27-cm ___ 28-cm ___ I Don't Know ___ No Response ___

Transitivity of Weight

Materials:

One red and one gray clay ball of equal weight
One gray clay ball of a lighter weight

Instructions:

The E places the three clay balls in the middle of the table 8-10 inches from the S, and says:

HERE ARE SOME CLAY BALLS WE WILL BE WORKING WITH.

The E then hands the S one red and one gray clay ball of equal weight. The S is required to verbalize this latter fact.

DO THESE TWO CLAY BALLS WEIGH THE SAME?

Yes ___ No ___ I Don't Know ___ No Response ___

Next, the E removes the gray clay ball from the S's hand and places the gray ball on the table 8-10 inches in front of the hand in which it was held. Then the red clay ball is removed and placed in the hand opposite the one in which it originally appeared. Next the lighter gray clay ball is placed in the remaining empty hand, so the S will know that the red ball is the heavier of the two. The S also is required to verbalize this latter fact.

DOES ONE OF THE CLAY BALLS WEIGH MORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(If "Yes," then) WHICH ONE?

Red ___ Gray ___ I Don't Know ___ No Response ___

Transitivity of Weight continued

The gray clay ball is removed and placed on the table 8-10 inches in front of the hand in which it was held. Finally, the E removes the red clay ball from the table, and asks the following questions:

(a) DO THESE TWO CLAY BALLS WEIGH THE SAME?

Yes ___ No ___ I Don't Know ___ No Response ___

(b) DOES ONE OF THE CLAY BALLS WEIGH MORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(If "Yes," then) WHICH ONE?

Heavy ___ Light ___

(c) DOES ONE OF THE CLAY BALLS WEIGH LESS?

Yes ___ No ___ I Don't Know ___ No Response ___

(If "Yes," then) WHICH ONE?

Light ___ Heavy ___

Conservation of Length

Identity Format

Materials:

One 28-cm string

Instructions:

(1.) Prediction: Placing the 28-cm string in the middle of the table 8-10 inches from the S, so the length runs horizontally in a straight line from the S's left to right, the E asks the following questions:

(a) IF I WERE TO MAKE THIS STRING INTO A CIRCLE, WOULD THE STRING STILL HAVE THE SAME LENGTH?

Yes ___ No ___ I Don't Know ___ No Response ___

(b) IF I WERE TO MAKE THIS STRING INTO A CIRCLE, WOULD THE STRING BE LONGER?

Yes ___ No ___ I Don't Know ___ No Response ___

(c) IF I WERE TO MAKE THIS STRING INTO A CIRCLE, WOULD THE STRING BE SHORTER?

Yes ___ No ___ I Don't Know ___ No Response ___

Conservation of Length continued

(2.) Deformation: The E then forms the string into a circle (toward the S), and asks the following questions:

(a) IS THIS STRING THE SAME LENGTH AS BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(b) IS THIS STRING LONGER THAN BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(c) IS THIS STRING SHORTER THAN BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

Conservation of Length

Equivalence Format

Materials:

Two 28-cm strings

Instructions:

The E places the two strings side-by-side in the middle of the table 8-10 inches from the S, so the length runs horizontally from the S's left to right, and so the strings are observed to be of equal length. The S is required to verbalize this latter fact.

ARE THESE TWO STRINGS THE SAME LENGTH?

Yes ___ No ___ I Don't Know ___ No Response ___

(1.) Prediction: Leaving the strings exactly as they are while pointing to the string nearest the S, the E asks the following questions:

(a) IF I WERE TO MAKE THIS STRING INTO A CIRCLE, WOULD THE TWO STRINGS STILL HAVE THE SAME LENGTH?

Yes ___ No ___ I Don't Know ___ No Response ___

(b) IF I WERE TO MAKE THIS STRING INTO A CIRCLE, WOULD ONE OF THE STRINGS BE LONGER?

Yes ___ No ___ I Don't Know ___ No Response ___

(c) IF I WERE TO MAKE THIS STRING INTO A CIRCLE, WOULD ONE OF THE STRINGS BE SHORTER?

Yes ___ No ___ I Don't Know ___ No Response ___

Conservation of Length continued

(2.) Deformation: The E then forms the string nearest the S into a circle (toward the S), and asks the following questions:

(a) ARE THESE TWO STRINGS THE SAME LENGTH AS BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(b) IS ONE OF THE STRINGS LONGER THAN BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(c) IS ONE OF THE STRINGS SHORTER THAN BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

Conservation of Weight

Identity Format

Materials:

One green clay ball

Instructions:

(1.) Prediction: Placing the green clay ball in the middle of the table 8-10 inches from the S, the E asks the following questions:

(a) IF I WERE TO ROLL THIS CLAY BALL INTO A HOT DOG, WOULD THE PIECE OF CLAY STILL HAVE THE SAME WEIGHT?

Yes ___ No ___ I Don't Know ___ No Response ___

(b) IF I WERE TO ROLL THIS CLAY BALL INTO A HOT DOG, WOULD THE PIECE OF CLAY WEIGH MORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(c) IF I WERE TO ROLL THIS CLAY BALL INTO A HOT DOG, WOULD THE PIECE OF CLAY WEIGH LESS?

Yes ___ No ___ I Don't Know ___ No Response ___

Conservation of Weight continued

(2.) Deformation: The E then rolls the clay ball into a hot dog, and asks the following questions:

(a) DOES THIS PIECE OF CLAY WEIGH THE SAME AS BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(b) DOES THIS PIECE OF CLAY WEIGH MORE THAN BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(c) DOES THIS PIECE OF CLAY WEIGH LESS THAN BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

Conservation of Weight

Equivalence Format

Materials:

Two brown clay balls of equal weight

Instructions:

The E gives the S a clay ball to hold in each hand so the balls are observed to be of equal weight. The S is required to verbalize this latter fact.

ARE THESE TWO BALLS THE SAME WEIGHT?

Yes ___ No ___ I Don't Know ___ No Response ___

(1.) Prediction: Taking the balls from the S and placing them on the table side-by-side 8-10 inches from the S, the E asks the following questions while pointing to one of the stimuli:

(a) IF I WERE TO FLATTEN THIS CLAY BALL INTO A PANCAKE, WOULD THE TWO PIECES OF CLAY STILL HAVE THE SAME WEIGHT?

Yes ___ No ___ I Don't Know ___ No Response ___

(b) IF I WERE TO FLATTEN THIS CLAY BALL INTO A PANCAKE, WOULD ONE OF THE PIECES OF CLAY WEIGH MORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(c) IF I WERE TO FLATTEN THIS CLAY BALL INTO A PANCAKE, WOULD ONE OF THE PIECES OF CLAY WEIGH LESS?

Yes ___ No ___ I Don't Know ___ No Response ___

Conservation of Weight continued

(2.) Deformation: The E then flattens the clay ball into a pancake, and asks the following questions:

(a) DO THESE TWO PIECES OF CLAY WEIGH THE SAME AS BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(b) DOES ONE OF THE PIECES OF CLAY WEIGH MORE THAN BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

(c) DOES ONE OF THE PIECES OF CLAY WEIGH LESS THAN BEFORE?

Yes ___ No ___ I Don't Know ___ No Response ___

Groupement I

E reads the questions very slowly and emphasizes the words underlined. E may repeat questions only.

I. Circular Stimulus

A. Preliminary Counting.

1. COUNT ALL THE CIRCLES THAT HAVE SOME YELLOW ON THEM. (4)*

2. COUNT ALL THE CIRCLES THAT DON'T HAVE YELLOW ON THEM. (4)*

B. Composition.

1. ARE THERE THE SAME NUMBER OF CIRCLES WITH YELLOW ON THEM AS THERE ARE CIRCLES?

2. ARE THERE MORE CIRCLES THAN THERE ARE CIRCLES WITH YELLOW ON THEM?

C. Inversion.

1. IF I TOOK AWAY THE CIRCLES WITH YELLOW ON THEM WOULD THERE BE SOME CIRCLES LEFT?

2. IF I TOOK AWAY THE CIRCLES WITH YELLOW ON THEM WOULD ALL THE CIRCLES BE GONE?

II. Triangular Stimulus

A. Preliminary Counting.

1. COUNT ALL THE TRIANGLES THAT HAVE SOME YELLOW ON THEM. (4)*

2. COUNT ALL THE TRIANGLES THAT DON'T HAVE YELLOW ON THEM. (4)*

B. Composition.

1. ARE THERE THE SAME NUMBER OF TRIANGLES WITH YELLOW ON THEM AS THERE ARE TRIANGLES?

2. ARE THERE MORE TRIANGLES THAN THERE ARE TRIANGLES WITH YELLOW ON THEM?

C. Inversion.

1. IF I TOOK AWAY THE TRIANGLES WITH YELLOW ON THEM WOULD THERE BE SOME TRIANGLES LEFT?

2. IF I TOOK AWAY THE TRIANGLES WITH YELLOW ON THEM WOULD ALL THE TRIANGLES BE GONE?

* E may help S obtain correct number of each stimulus.

Groupement II

E reads the questions very slowly and emphasizes the words underlined. E may repeat questions only.

I. Circular Stimulus.

A. Preliminary Counting.

1. COUNT ALL THE CIRCLES THAT HAVE SOME RED ON THEM. (4)*

2. COUNT ALL THE CIRCLES THAT DON'T HAVE RED ON THEM. (4)*

B. Composition.

1. ARE THERE THE SAME NUMBER OF CIRCLES WITH RED ON THEM AS THERE ARE CIRCLES?

2. ARE THERE MORE CIRCLES THAN THERE ARE CIRCLES WITH RED ON THEM?

C. Inversion.

1. IF I TOOK AWAY THE CIRCLES WITH RED ON THEM, WOULD THERE BE SOME CIRCLES LEFT?

2. IF I TOOK AWAY THE CIRCLES WITH RED ON THEM, WOULD ALL THE CIRCLES BE GONE?

II. Triangular Stimulus.

A. Preliminary Counting.

1. COUNT ALL THE TRIANGLES THAT HAVE SOME RED ON THEM. (4)*

2. COUNT ALL THE TRIANGLES THAT DON'T HAVE RED ON THEM. (4)*

B. Composition.

1. ARE THERE THE SAME NUMBER OF TRIANGLES WITH RED ON THEM AS THERE ARE TRIANGLES?

2. ARE THERE MORE TRIANGLES THAN THERE ARE TRIANGLES WITH RED ON THEM?

C. Inversion

1. IF I TOOK AWAY THE TRIANGLES WITH RED ON THEM, WOULD THERE BE SOME TRIANGLES LEFT?

2. IF I TOOK AWAY THE TRIANGLES WITH RED ON THEM, WOULD ALL THE TRIANGLES BE GONE?

* E may help S obtain correct number of each stimulus.

Groupement III

E reads the question very slowly and emphasizes the words underlined. E may repeat questions only.

I. Circular Stimulus.

A. Preliminary Counting.

1. COUNT ALL THE CIRCLES THAT HAVE SOME RED ON THEM. (4)*

2. COUNT ALL THE CIRCLES THAT HAVE SOME YELLOW ON THEM. (4)*

B. Composition.

1. ARE THERE THE SAME NUMBER OF CIRCLES WITH RED ON THEM AS THERE ARE CIRCLES WITH BOTH RED AND YELLOW ON THEM?

2. ARE THERE MORE CIRCLES WITH RED ON THEM THAN THERE ARE CIRCLES WITH BOTH RED AND YELLOW ON THEM?

C. Inversion.

1. IF I TOOK AWAY THE CIRCLES WITH RED ON THEM, WOULD THERE BE ANY CIRCLES WITH BOTH RED AND YELLOW ON THEM LEFT?

2. IF I TOOK AWAY THE CIRCLES WITH RED ON THEM, WOULD ALL THE CIRCLES WITH BOTH RED AND YELLOW ON THEM BE GONE?

II. Triangular Stimulus.

A. Preliminary Counting.

1. COUNT ALL THE TRIANGLES THAT HAVE SOME RED ON THEM. (4)*

2. COUNT ALL THE TRIANGLES THAT HAVE SOME YELLOW ON THEM. (4)*

B. Composition.

1. ARE THERE THE SAME NUMBER OF TRIANGLES WITH YELLOW ON THEM AS THERE ARE TRIANGLES WITH BOTH RED AND YELLOW ON THEM?

2. ARE THERE MORE TRIANGLES WITH YELLOW ON THEM THAN THERE ARE TRIANGLES WITH BOTH RED AND YELLOW ON THEM?

C. Inversion.

1. IF I TOOK AWAY THE TRIANGLES WITH YELLOW ON THEM WOULD THERE BE ANY TRIANGLES WITH BOTH RED AND YELLOW ON THEM LEFT?

2. IF I TOOK AWAY THE TRIANGLES WITH YELLOW ON THEM, WOULD ALL THE TRIANGLES WITH BOTH RED AND YELLOW BE GONE?

* E may help S obtain correct number of each stimulus.

Groupement IV

E reads questions very slowly and emphasizes words underlined. E may repeat questions only.

I. Stimulus 1.

A. Preliminary Counting.

1. COUNT ALL THE YELLOW THINGS. (4)*

2. COUNT ALL THE YELLOW CIRCLES. (2)*

B. Composition.

1. ARE THERE THE SAME NUMBER OF YELLOW CIRCLES AS YELLOW THINGS?

2. ARE THERE MORE YELLOW THINGS THAN YELLOW CIRCLES?

C. Inversion.

1. IF I TOOK AWAY THE YELLOW THINGS, WOULD THERE BY ANY YELLOW CIRCLES LEFT?

2. IF I TOOK AWAY THE YELLOW THINGS WOULD ALL THE YELLOW CIRCLES BE GONE?

II. Stimulus 2.

A. Preliminary Counting.

1. COUNT ALL THE YELLOW THINGS. (4)*

2. COUNT ALL THE TRIANGLES. (2)*

B. Composition.

1. ARE THERE THE SAME NUMBER OF YELLOW TRIANGLES AS YELLOW THINGS?

2. ARE THERE MORE YELLOW THINGS THAN YELLOW TRIANGLES?

C. Inversion.

1. IF I TOOK AWAY THE YELLOW THINGS, WOULD THERE BY ANY YELLOW TRIANGLES LEFT?

2. IF I TOOK AWAY THE YELLOW THINGS, WOULD ALL THE YELLOW TRIANGLES BE GONE?

* E may help S obtain correct number of each stimulus.

Groupement V

Like transitivity--the sticks are separated by 2 ft. The middle stick is brought to each side and compared.
E reads the question very slowly and emphasizes the words underlined.
E may repeat questions only.

I. Length

A. Preliminary Comparisons.

1. E shows S that the Blue stick is shorter than the Green stick.*

2. E shows S that the Green stick is shorter than the Red stick.*

B. Composition.

1. ARE THE BLUE AND RED STICKS THE SAME LENGTH?

2. IS THE BLUE STICK SHORTER THAN THE RED STICK?

C. Reciprocity Comparisons.

1. E shows S that the Red stick is longer than the Green stick.*

2. E shows S that the Green stick is longer than the Blue stick.*

D. Reciprocity.

1. ARE THE RED AND BLUE STICKS THE SAME LENGTH?

2. IS THE RED STICK LONGER THAN THE BLUE STICK?

II. Weight

A. Preliminary Comparisons.

1. E shows S that the Red stick is lighter than the Green stick.*

2. E shows S that the Green stick is lighter than the Blue stick.*

B. Composition.

1. DO THE RED AND BLUE STICKS WEIGH THE SAME?

2. IS THE RED STICK LIGHTER THAN THE BLUE STICK?

C. Reciprocity Comparisons.

1. E shows S that the Blue stick is heavier than the Green

Groupement V continued

stick.*

2. E shows S that the Green stick is heavier than the Red stick.*

D. Reciprocity.

1. DO THE BLUE AND RED STICKS WEIGH THE SAME?

2. IS THE BLUE STICK HEAVIER THAN THE RED STICK?

* E first asks S what is the relationship between the two stimuli. E helps S to understand and verbalize the relationship before going on, i.e., ARE THESE THE SAME? HOW ARE THEY DIFFERENT? WHICH ONE IS LONGER (SHORTER, HEAVIER, LIGHTER)?.

Groupement VI

E reads questions very slowly and emphasizes the words underlined. E may repeat questions only.

I. Length

A. Preliminary Comparisons.

1. E shows S that the Blue and [illegible] sticks are the same length.*

2. E shows S that the Green and Red sticks are the same length.*

B. Composition.

1. ARE THE BLUE AND RED STICKS THE SAME LENGTH?

2. IS THE BLUE STICK SHORTER THAN THE RED STICK?

C. Reciprocity Comparisons.

1. E shows S that the Red and Green sticks are the same length.*

2. E shows S that the Green and Blue sticks are the same length.*

D. Reciprocity.

1. ARE THE RED AND BLUE STICKS THE SAME LENGTH?

2. IS THE RED STICK LONGER THAN THE BLUE STICK?

II. Weight

A. Preliminary Comparisons.

1. E shows S that the Red and Green sticks weigh the same.*

2. E shows S that the Green and Blue sticks weigh the same.*

B. Composition.

1. DO THE RED AND BLUE STICKS WEIGH THE SAME?

2. IS THE RED STICK LIGHTER THAN THE BLUE STICK?

C. Reciprocity Comparisons.

1. E shows S that the Blue and Green sticks weigh the same.*

2. E shows S that the Green and Red sticks weigh the same.*

Groupement VI continued

D. Reciprocity.

1. DO THE BLUE AND RED STICKS WEIGH THE SAME?

2. IS THE BLUE STICK HEAVIER THAN THE RED STICK?

* E first asks S what is the relationship between the two stimuli. E helps S to understand and verbalize the relationship before going on, i.e., ARE THESE THE SAME? HOW ARE THEY DIFFERENT? WHICH ONE IS LONGER (SHORTER, HEAVIER, LIGHTER)?.

Groupement VII

E reads questions very slowly and emphasizes the words underlined. E may repeat questions only.

Length and Weight

I. Preliminary Comparisons.

A. E shows S that the Red stick is both shorter and lighter than the Blue stick.*

B. E shows S that the Blue stick is both shorter and lighter than the Green stick.*

II. Composition.

A. ARE THE RED AND GREEN STICKS THE SAME LENGTH?

B. DO THE RED AND GREEN STICKS WEIGH THE SAME?

C. IS THE RED STICK SHORTER THAN THE GREEN STICK?

D. IS THE RED STICK LIGHTER THAN THE GREEN STICK?

III. Reciprocity Comparisons.

A. E shows S that the Green stick is both longer and heavier than the Blue stick.*

B. E shows S that the Blue stick is both longer and heavier than the Red stick.*

IV. Reciprocity.

A. ARE THE GREEN AND RED STICKS THE SAME LENGTH?

B. DO THE GREEN AND RED STICKS WEIGH THE SAME?

C. IS THE GREEN STICK LONGER THAN THE RED STICK?

D. IS THE GREEN STICK HEAVIER THAN THE RED STICK?

* E first asks S what is the relationship between the two stimuli. E helps S to understand and verbalize the relationship before going on, i.e., ARE THESE THE SAME? HOW ARE THEY DIFFERENT? WHICH ONE IS LONGER (SHORTER, HEAVIER, LIGHTER)?.

Groupement VIII

E reads questions very slowly and emphasizes the words underlined. E may repeat questions only.

I. Preliminary Comparisons.

A. E shows S that the Green stick is shorter and the same weight as the Red stick.*

B. E shows S that the Red stick is shorter and the same weight as the Blue stick.*

II. Composition.

A. ARE THE GREEN AND BLUE STICKS THE SAME LENGTH?

B. DO THE GREEN AND BLUE STICKS WEIGH THE SAME?

C. IS THE GREEN STICK SHORTER THAN THE BLUE STICK?

D. IS THE GREEN STICK LIGHTER THAN THE BLUE STICK?

III. Reciprocity Comparisons.

A. E shows S that the Blue stick is longer and weighs the same as the Red stick.*

B. E shows S that the Red stick is longer and weighs the same as the Green stick.*

IV. Reciprocity.

A. ARE THE BLUE AND GREEN STICKS THE SAME LENGTH?

B. DO THE BLUE AND GREEN STICKS WEIGH THE SAME?

C. IS THE BLUE STICK LONGER THAN THE GREEN STICK?

D. IS THE BLUE STICK HEAVIER THAN THE GREEN STICK?

* E first asks S what is the relationship between the two stimuli. E helps S to understand and verbalize the relationship before going on, i.e., ARE THESE THE SAME? HOW ARE THEY DIFFERENT? WHICH ONE IS LONGER (SHORTER, HEAVIER, LIGHTER)?.

Class Inclusion B

1. Materials: 3 red and 2 blue triangles

   ARE THERE MORE TRIANGLES OR MORE RED FIGURES?

   more triangles*, more red figures, other

2. Materials: 3 yellow and 2 blue circles

   ARE THERE MORE BLUE FIGURES OR MORE CIRCLES?

   more circles*, more blue figures, other

3. Materials: 3 red and 2 blue triangles, 4 blue circles

   a. ARE THERE MORE TRIANGLES OR MORE RED FIGURES?

      more triangles*, more red figures, other

   b. ARE THERE MORE BLUE FIGURES OR MORE CIRCLES?

      more blue figures*, more circles, other

   c. ARE THERE MORE BLUE FIGURES OR MORE TRIANGLES?

      more blue figures*, more triangles, other

* correct response

APPENDIX B

MONOTONE RELATIONSHIONSHIP OF GAMMA AND BENTLER'S COEFFICIENT

Haertel, E. A proof that for any two 4-fold tables, Bentler's Coefficient and γ will be similarlarly ranked, unpublished proof, University of Wisconsin June 18, 1974.

Lemma: $0 \leq p < q \Rightarrow \frac{p-1}{p+1} < \frac{q-1}{q+1}$

Proof:

$$p < q$$
$$-q < -p$$
$$p + (-q) < q + (-p)$$
$$pq - q + p - 1 < pq - p + q - 1$$
$$(p-1)(q+1) < (q-1)(p+1)$$
$$\frac{(p-1)(q+1)}{(p+1)(q+1)} < \frac{(q-1)(p+1)}{(p+1)(q+1)}$$
$$\frac{p-1}{p+1} < \frac{q-1}{q+1}$$

q.e.d.

Let γ_{B1} and γ_1 be taken from the table

a_1	b_1
c_1	d_1

Let γ_{B2} and γ_2 be taken from the table

a_2	b_2
c_2	d_2

$$\gamma = \frac{ad - bc}{ad + bc}, \quad \gamma_B = \frac{ad - bc}{ad + bc + 2\sqrt{abcd}}$$

By inspection, each coefficient attains its maximum if and only if $bc = 0$. Hence if $b_1c_1 = 0$ and/or $b_2c_2 = 0$, the assertion is proved.

Else, let $x_1 = \frac{a_1b_1}{b_1c_1}$, $x_2 = \frac{a_2d_2}{b_2c_2}$

Note that $0 \leq x_1$, $0 \leq x_2$

$$\gamma = \frac{ad - bc}{ad + bc}$$

$$= \frac{\frac{ad}{bc} - \frac{bc}{bc}}{\frac{ad}{bc} + \frac{bc}{bc}}$$

$$= \frac{x - 1}{x + 1}$$

$$\gamma_B = \frac{ad - bc}{ad + bc + 2\sqrt{abcd}}$$

$$= \frac{\frac{ad}{bc} - \frac{bc}{bc}}{\frac{ad}{bc} + \frac{bc}{bc} + 2\sqrt{\frac{abcd}{(bc)^2}}}$$

$$= \frac{x - 1}{x + 1 + 2\sqrt{x}}$$

$$= \frac{(\sqrt{x} + 1)(\sqrt{x} - 1)}{(\sqrt{x} + 1)(\sqrt{x} + 1)}$$

$$= \frac{\sqrt{x} - 1}{\sqrt{x} + 1}$$

Thus, if $x_1 = x_2$ then $\gamma_{B1} = \gamma_{B2}$ and $\gamma_1 = \gamma_2$, hence both rankings are tied.

If $x_1 \neq x_2$, without loss of generality assume $x_1 < x_2$

Then by lemma, $\gamma_1 < \gamma_2$.

$x_1 < x_2 \Rightarrow \sqrt{x_1} < \sqrt{x_2}$ (over the domain R^+)

Thus by lemma, $\gamma_{B1} < \gamma_{B2}$ and the rankings agree.

Corollary: For any 4-fold table if $\gamma < 1$

$$\frac{1 + \gamma}{1 - \gamma} = \left(\frac{1 + \gamma_B}{1 - \gamma_B}\right)^2$$

Thus given either statistic, the value of the other may be obtained.

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The end